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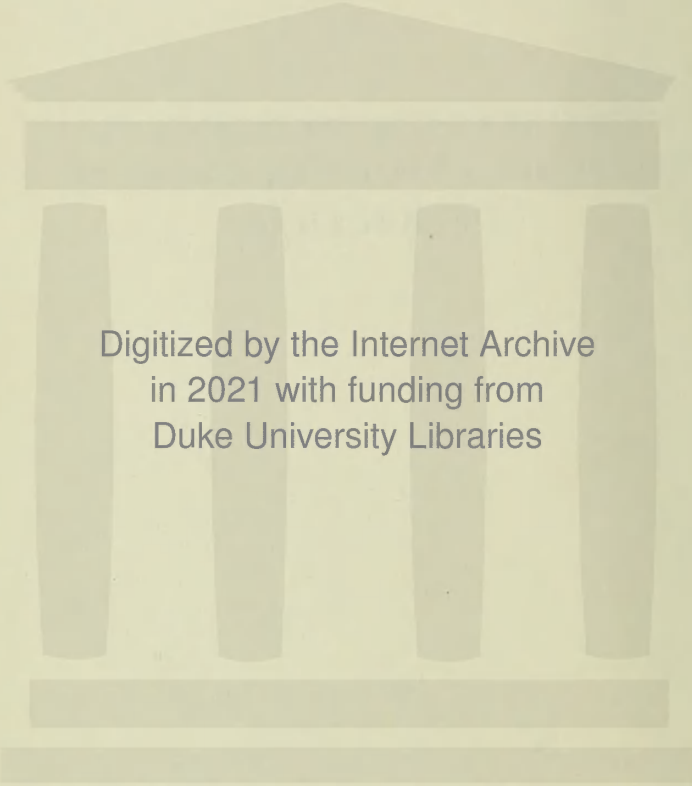








MEANINGFUL vs. MECHANICAL  
LEARNING: A STUDY IN GRADE III  
SUBTRACTION



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# MEANINGFUL vs. MECHANICAL LEARNING: A STUDY IN GRADE III SUBTRACTION

BY

WILLIAM A. BROWNELL

AND

HAROLD E. MOSER

with the assistance of

SAMUEL L. ABBOTT, JR., KATHRYN FRAZIER,

MARY F. KARR, J. H. OSTWALT, AND

JAMES F. WEAVER



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DEDICATED TO THE MEMORY OF  
CHARLES HUBBARD JUDD,  
TO WHOM TEACHERS, NOT ALONE OF ARITHMETIC,  
OWE A FAR GREATER DEBT THAN THEY  
MAY EVER KNOW

76977



## ACKNOWLEDGMENTS

The investigation described in this monograph was made in the year 1942-43. A multiplicity of activities in the "war years" prevented earlier publication of the results. The regrettable lapse of time between completion of the study and announcement of findings has not in any known way impaired the validity of the data or their significance.

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Miss Elizabeth Boyd  
Mrs. G. W. Davis  
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Mr. Harold E. Moser

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WILLIAM A. BROWNELL

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MEANINGFUL vs. MECHANICAL  
LEARNING: A STUDY IN GRADE III  
SUBTRACTION



## CHAPTER I

### THE PROBLEM AND ITS BACKGROUND

This monograph, ostensibly a contribution to educational practice, is, it is hoped, not without psychological significance as well. The educational, or practical, problem which has been studied has to do with the relative merits of two different procedures for subtracting in such examples as those in the box. In these examples the first subtractions ( $1 - 9$  and  $0 - 8$ ) are impossible without first changing the 1 of 81 to 11 and the 0 of 340 to 10 and then changing the figures in the tens' places also. Two procedures for performing such subtractions (to be described below) were taught, each in two different ways: (1) as mechanical skills and (2) as rational activities. From this difference in instruction arise questions of psychological or theoretical interest, centered around the issue of meaning in such learning tasks.

|     |      |
|-----|------|
| 81  | 340  |
| -49 | -218 |
| 32  | 122  |

Obviously the two aspects of the problem, educational and psychological, are so closely interrelated that only arbitrarily can they be separated. And this is as it should be, for any educational investigation of learning (here, the learning or arithmetical processes) is after all an investigation of learning, and learning is one of the major concerns of psychology.

#### SUBTRACTION PROCEDURES

To the layman it may be something of a surprise to learn that 49 can be subtracted from 81 in a variety of ways. A few, but just a few, of the possible thought patterns follow:

|     |
|-----|
| 81  |
| -49 |
| 32  |

- (1) Nine and what is 11? 2; write 2. Four and what is 7? 3; write 3.
- (2) Nine from 11, 2; write 2. Five from 8, 3; write 3.
- (3) Nine and 1 is 10; 1 (from  $11 - 10$ ) and 1 is 2; write 2. Five and 3 is 8; write 3.
- (4) Nine from 11, 2; write 2. Four from 7, 3; write 3.
- (5) Nine and what is 11? 2; write 2. Five and what is 8? 3; write 3.

A comparison of these thought patterns reveals differences (a) *in the way in which each subtraction is performed*—additively, as

(1), (3), and (5); or subtractively as in (2) and (4); (b) *in the way in which the minuend ones' figure is changed to 11*—by adding 10 ones at once, as in (1), (2), (4), and (5); or by building it first to 10 (3), in what is known as the Complementary (or Decimal Complementary) Method; and (c) *in the way in which the tens' figures in the minuend and subtrahend are dealt with*—by decreasing the 8 of 81 to 7, to get the 10 ones, as in (1) and (4), according to the Decomposition Method, or by increasing the subtrahend tens' figure, 4, at the time that the 10 ones are added to the minuend ones' figure, according to the Equal Additions Method, as in (2), (3), and (5) above.

There are other ways also in which 49 may be subtracted from 81; but enough ways have been exemplified to show the layman, who is perhaps familiar only with the procedure which he himself uses, that subtraction methods do differ and that therefore there must be some question concerning the comparative values of the various procedures.

There is this question, and it has engaged the interest of research workers and students of arithmetic for thirty years and more. The research reports and theoretical discussions have been ably analyzed and critically summarized at several different times—by Osburn (19)<sup>1</sup> in 1927, by Sister Irmira (6) in 1928, by Ruch and Mead (23) in 1930, by Monroe and Engelhart (16) in 1931, and most recently by Johnson (10) in 1938 and by Murray (18) in 1941. The number and quality of these periodic syntheses make it unnecessary here to rehearse again the details of experimental procedure and findings. Instead, we may turn at once to the specific problems of the present research.

#### THE SUBTRACTION PROCEDURES INVESTIGATED IN THIS STUDY

There are those who advocate that subtraction in such examples as  $972 - 54$  should be taught by the Complementary Method or by the Austrian Method (see p. 6) or by some combination of methods. To support their positions they have some research evidence and a number of logical arguments.<sup>2</sup> But these individuals have not made much headway in this country, and the battle has resolved itself into the relative advantages

|       |
|-------|
| 972   |
| $-54$ |
| 918   |

<sup>1</sup> Numbers of this kind identify the published reports in the chapter bibliography beginning on page 23.

<sup>2</sup> See Johnson (10, chaps. ii and iii) for an interesting and valuable statement of the history of subtraction procedures and of the claims advanced for each. Fuller expositions will be found in 24 and 25.

of Equal Additions (hereinafter designated EA) and of Decomposition (D). Of these two procedures, the former is favored in the schools of Great Britain and the latter in the schools of this country. Moreover, the question as to whether children should think each subtraction additively ( $12 - 4$  becomes  $4 + ? = 12$ ) or subtractively ( $12 - 4 = ?$ ) has been settled, at least in practice (there is little relevant research), in favor of the subtraction method.

Accordingly, by this rather cavalier manner of delimiting the problem, it is possible to say that the question now largely at stake is whether children shall be taught subtraction in examples of the kind already illustrated by Subtractive Equal Additions (SEA) or by Subtractive Decomposition (SD); and since in both instances the diminishing is done by subtractive methods, the problem can be still further limited to become: EA or D, which? (the subtraction process being assumed in both instances).

According to EA the child is taught (at once or later on) to think merely: "Four from 12, 8; write 8; 6 from 7, 1; write 1."

|     |
|-----|
| 72  |
| -54 |
| 18  |

According to D, the child, sooner or later, thinks: "Four from 12, 8; write 8; 5 from 6, 1; write 1."

The term "equal additions" is appropriate for the first procedure because a ten is added to both terms in the example, as 10 ones to the 2 of 72, and as 1 ten to the 5 of 54. Likewise, "decomposition" is appropriate for the second procedure, because the minuend, 72, is "decomposed" (made equivalent) to  $60 + 12$ .

#### RELATED RESEARCH

As a matter of fact, one almost needs to apologize for reporting another investigation dealing with the comparative merits of EA and D as here defined. There are in the literature already at least fifteen studies relating exclusively or largely to this problem. Moreover, there is an overwhelming agreement in the experimental data in favor of EA, which has usually been found to be both faster and more accurate than D. Then, why another research study? The answer to this question and the occasion for continued research will be clearer if attention is first directed to the two most recent, most elaborate, and most carefully done investigations.

#### *The Johnson Study (10)*

Johnson has had a long-time interest in the issues with which the present research deals. In 1924 he made his first large-scale

attack upon the problem (7), publishing a second study (9) in 1931 and a third (10) in 1938. While in each case he included subtraction procedures other than D and EA, he consistently found the advantage, as between the two, to lie with EA. It is the last of his three studies with which we are here primarily concerned. In this, as in his first two studies, he employed the experimental technique of differential testing.

In this last investigation Johnson made use of about 1200 subjects drawn from eight schools which could be classified according to the subtraction procedure which had been emphasized for a period of years. These procedures are illustrated (10, p. 47) as follows:

| METHOD D |               | METHOD E (here EA) |               | METHOD A <sup>3</sup> |              |
|----------|---------------|--------------------|---------------|-----------------------|--------------|
| 82       | 7 from 12 = 5 | 82                 | 7 from 12 = 5 | 82                    | 7 and 5 = 12 |
| - 37     | 3 from 7 = 4  | - 37               | 4 from 8 = 4  | - 37                  | 4 and 4 = 8  |
| 45       |               | 45                 |               | 45                    |              |
| 600      | 6 from 10 = 4 | 600                | 6 from 10 = 4 | 600                   | 6 and 4 = 10 |
| -146     | 4 from 9 = 5  | -146               | 5 from 10 = 5 | -146                  | 5 and 5 = 10 |
|          | 1 from 5 = 4  |                    | 2 from 6 = 4  |                       | 2 and 4 = 6  |
| 454      |               | 454                |               | 454                   |              |

Johnson took the unusual, but essential, precaution of determining the procedures his subjects actually used, whatever the procedure they were supposed to have learned. He had each subject "record on a separate sheet of paper" just what he said (thought) when he subtracted in the two examples  $82 - 37$  and  $600 - 146$ . He then classified his subjects according to procedure, and equated his three experimental groups on the bases: MA, IQ (McCall Multi-Mental Scale), and number of errors and time required on two preliminary tests on the 100 simple subtraction combinations. His groups thus formed comprised 526 D-subjects, 342 E-subjects, and 186 A-subjects ranging from Grade 3 through Grade 8. (For some reason forty-three adult subjects were included in his D- and his E-groups in certain comparisons.)

*Experimental Results.*—Johnson rests his case on data obtained from his Test 2, of which there were two forms, both taken by all three groups of subjects. Each of these test forms contained thirty-two examples, in which all subtraction combinations occurred once. To determine the relative merits of the three subtraction procedures in the matter of *accuracy*, Johnson noted the increase in average

<sup>3</sup> Commonly known as the Austrian Method.

number of errors in Test 2 over the average number in Test 1 (simple combinations). The increases in error were: D, 5.73; E, 4.84; A, 4.92. The D-subjects obviously made the poorest showing, with little to choose between the E- and the A-subjects. The Critical Ratios<sup>4</sup> were: D vs. E, 3.61; D vs. A, 2.79; E vs. A, 0.271; D vs. E and A combined, 3.86.

To determine the differences in *rate* of work Johnson noted the increase in average time from Test 1 to Test 2. Here the mean differences were: D, 3.19; E, 2.76; A, 1.91; E and A combined, 2.48 (minutes in all cases). The Critical Ratios were: D vs. E, 4.25; D vs. A, 13.6; E vs. A, 8.3; D vs. E and A combined, 8.1.

Subsequently comparisons were made with respect to *time* alone between (a) groups of D- and of E- and A-subjects who made no errors on Test 1 or on Test 2; (b) similar groups who made no errors on Test 1 and 1 to 4 errors on Test 2, and (c) similar groups who made no errors on Test 1 and 0 to 4 errors on Test 2. In all instances the D-Subjects were poorest, and by very large Critical Ratios.

From his data Johnson drew the conclusion that E is to be preferred to D as a subtraction procedure, his criteria being those of proficiency in skill, namely, accuracy and rate of work. To Johnson these criteria were sufficient, and he accordingly proposed that American schools abandon D in favor of E, since there is small likelihood that they will adopt A in spite of the fact that it is as accurate as E and much more rapid.

*Logical Evidence.*—In his "Pedagogical Implications" (beginning on p. 67) Johnson advances logical evidence to support his position, his points being: (a) E (or EA, to use the terminology of this report) makes a lighter demand upon memory than does D, since the "memory element" in changing the subtrahend figures in EA after the first subtraction is less difficult than that in changing the minuend figures in D; (b) EA is easier than D when two or more successive zeros occur in the minuend (e.g., 4000-2795); (c) in the example 81 - 65, one addition and three subtractions are required by D, whereas two additions and two subtractions are required by EA, and it is well known that children find addition easier than subtraction.

<sup>4</sup> The formula used is not stated. In his earlier experiments Johnson employed the so-called "short formula" for computing the P. E. diff.

$$\text{P. E. diff. } M_D - M_E = \sqrt{\text{P. E. } M_D^2 + \text{P. E. } M_E^2}$$

With regard to the comparative ease of rationalization, "a cause of argument for many years among teachers," Johnson says (p. 70) :

Advocates of the decomposition method hold that their method is more easily explained, while advocates of the equal additions plan point out that experimental evidence is all on their side. There has been no scientific evidence that the decomposition method is more easily explained. This judgment is merely the opinion of teachers who have been taught that method and hence naturally understand that method better and consequently can explain it better. In all probability the equal additions method can be explained equally well, but most teachers have not yet learned how best to explain it.

... let it be granted for the sake of argument that the decomposition method can be more easily rationalized for your pupils than the equal additions method. The question then resolves itself: Shall a method that has been proved to be least efficient after it is learned continue to be taught just because it is more easily explained upon first presentation? Let the reader be the judge.

*Criticism of the Johnson Experiment.*—Good as Johnson's study is, it is, nevertheless, open to several criticisms, four of which will be mentioned below.

(a) SUBJECTS. No information is given with respect to the grade level of Johnson's subjects. In Table I (p. 50), in which certain correlation coefficients are reported, 31 of the 440 subjects (7 per cent) are designated as having been drawn from Grades 3 and 4. If the relative grade distribution of subjects for the study as a whole is the same as, or like that of, the sample in the table, it is clear that the bulk of Johnson's data was obtained from children who were considerably removed from the time of initial learning. That is to say, on the hypothesis stated, Johnson measured the final proficiency of his subjects some years after they were introduced to the process of subtraction, and his report tells us nothing about the comparative ease of *learning* D and EA. It is not impossible that *at the time of learning* D may actually be easier than EA, the advantage disappearing with accumulated practice.

If Johnson had adequate data for Grade 3 and for Grade 4 it is unfortunate that he did not publish them separately, to show the comparative difficulty of the two subtraction procedures from grade to grade. If research should show that *initially* D can be taught much more successfully than can EA, then the evidence for EA must be discounted. Meanwhile, we have no assurance in Johnson's data, where measures for all grades are combined, that EA is easier to teach

and to learn in the lower grades, however much it may surpass D in efficiency in the higher grades.

(b) **ACCURACY DATA.** As has already been stated, Johnson claims superiority for EA because, with the subjects tested, it was quicker and more accurate. The data on accuracy are none too convincing in spite of the Critical Ratios reported. Johnson measured accuracy negatively, that is, in terms of errors, the average number for D-pupils being 5.73 on Test 2, in excess of the average number on Test 1. But equality of work can also be expressed positively in terms of correctness. When this is done, the per cents of success (if the errors on Test 1 are disregarded) are seen to be very much alike: D, 94.3; EA, 95.2; and A, 95.1. The greatest difference, between D and EA, amounts to about one subtraction in a hundred, a difference which, however reliable, is too small to be of much significance educationally. Apparently Johnson recognized this fact, for, quite correctly, he put the greater emphasis upon differences in time.<sup>5</sup> There can be no question that, for the subjects tested (it is necessary to repeat this limitation), EA proved to be faster than D.

(c) **EFFICIENCY AS THE SOLE CRITERION.** Johnson's criterion for evaluating EA and D is that of efficiency, as represented in measures of rate and accuracy of work. No one can question the value of these criterial measures: rate and accuracy *are* important. But one may question efficiency as the sole criterion, and the writers do raise this question. Two other criteria, of at least equal importance, have already been mentioned in passing: namely, (1) degree of insight which the subtraction procedures afford into the nature of the process and (2) the degree to which they facilitate learning at the time of initial instruction. There are still other criteria, such as: (3) the extent to which the two procedures enable the learner to profit from previous learning, (4) the extent to which they increase the chances of transfer to later aspects of the process, and (5) the extent to which they can be extended to cover related aspects of arithmetic. Still other criteria, or bases for evaluation, will be discussed toward the end of this chapter. It is sufficient here to note that on none of these five criteria just mentioned (and of the others to be mentioned) does the Johnson study supply data. On this account it would seem to be premature to decide the issue of relative worth in favor of EA merely because this procedure was found to

<sup>5</sup> Johnson has been consistent in basing his arguments for EA chiefly on measures of rate rather than of accuracy. See his first report (7), as well as his later defense of the same (8).

be faster and slightly more accurate with children in the intermediate and higher grades.

(d) THE EXPERIMENTAL TECHNIQUE OF DIFFERENTIAL TESTING. Johnson views with suspicion the research technique of "experiment-control groups" (10, pp. 41 ff.), because the "method is open to some grave weaknesses."

The chief one of these is the difficulty of keeping the experimental group on a comparable basis with the control group. The cause of this difficulty is that the experimental group is subjected to a more or less artificial treatment in subject-matter, method, teacher personnel, or a combination of these.

On this account Johnson employed the technique of differential testing. Its essence is this: two (or more) groups of subjects who have been exposed to differences in instruction are measured by means of tests, so designed as to bring out the differences, if any, in learning products. These differences are then ascribed to the differences in teaching. No control of instruction is undertaken; indeed, none could be undertaken without changing the technique to that of the controlled-group.

No one can question Johnson's statement that it is difficult to control teaching methods in experimental work; but it is doubtful whether the way out is to abandon the attempt in favor of differential testing. Moreover, the latter technique also has its weaknesses. Most of them arise from the circumstance which is regarded as its chief virtue, namely, absence of control of the teaching process. Knowingly or otherwise, those who employ the technique of differential testing operate under one or another of several assumptions: (1) all experimental groups have been taught equally well (or equally poorly); or, (2) differences in method of teaching, for this procedure or that, are nonexistent, or they produce no measurable differences in learning; or, (3) it is safe to assume that, with enough subjects and enough teachers, differences in teaching method are canceled out; or, (4) procedures, such as those which may be used in subtraction, prescribe arbitrarily their own peculiar and invariable methods of teaching: hence teaching can be disregarded as a factor, and the measures obtained may be taken as absolute measures.

Stated thus baldly, the assumptions will appeal to few educators or research workers. We know, for example, that the same procedure may be taught in a variety of ways. It is not impossible, for example, that EA taught one way is superior to D taught in a cer-

tain way, but that the balance may swing in favor of D if methods of teaching are changed. We can hardly wipe out the effects of differences in teaching by merely disregarding them, as is done when the technique of differential testing is employed. While we cannot control teaching methods rigidly and in detail, we can certainly exercise some control, enough to make sure that the supposed or assumed instructional methods are actual. The aim of research should be to determine the *best* instructional methods, or, as in the present instance, to test the comparative worth of the opposed subtraction procedures under conditions of teaching which are planned and supervised.

### *The Murray Study (18)*

Murray's findings, printed in a Scottish source, are not readily accessible to many readers in this country. More is the pity, for in the writers' opinion this investigation rates at the top in several respects. The report would be well worth thoughtful scrutiny if only for the excellent review of previous research with which it begins. Here in considerable detail are accounts of all published studies on subtraction, and not alone on the controversy regarding D and EA. Included are two reports (5, 12) which have not hitherto appeared in American bibliographies, the first of which is described by Murray (p. 37) as "one of the most extensive and most scientific applications of tests in the subject ever attempted." In the case of each investigation the essential facts of experimental technique and of findings are set forth, together with an evaluation which is acute and dispassionate.

In his own large-scale research Murray, like Johnson, made use of differential testing. Unlike Johnson, he subjected his data to elaborate statistical treatment (see below), and he reported separately data (a) for children who were in the early stages of learning their subtraction procedures (aged 8 to 9, in Primary II, the second year after the Infant Division) and (b) for children who were two years further along in their education (aged 10 to 11, in Primary IV). In connection with the latter feature it is pertinent to note one of Murray's critical comments on previous research (18, p. 41):

... many of the studies suffer from the serious defect that the tests were applied to pupils whose practice had become more or less habituated. It would seem desirable to secure full and reliable evidence in respect of the attainments of pupils at or about the initial learning stages, and to obtain similar evidence in respect of the effective permanency of skills relative to pupils several stages later in the school course.

Murray was interested in the comparative values of three subtraction procedures, those here designated as D (Subtractive Decomposition) and EA (Subtractive Equal Additions) and a third, C (Complementary Addition as distinguished from Decimal Complementary, to use Murray's terms). His procedure C is what has frequently been designated the "Austrian Method," and it is really a combination of additive subtraction and EA. (The complementary procedure can of course be used also with decomposition.) Children taught by C think for the example at the right: Eight + 6 = 14; write 6; carry 1. Three + 9 = 12; write 9; carry 1. Two + 1 = 3; write 1.

|                                                          |
|----------------------------------------------------------|
| $\begin{array}{r} 324 \\ -128 \\ \hline 196 \end{array}$ |
|----------------------------------------------------------|

Experimental subjects were drawn from fifty-five schools, selected so as to secure all types of educational environment, residential and industrial, rural and urban, and so on. The "junior subjects" (aged 8 to 9) totaled 1622: 316 C-subjects from twelve schools, 464 D-subjects from nineteen, and 842 EA-subjects from twenty-four. The 1675 in the "senior group" (aged 10 to 11) came from eleven C-schools (274 pupils), twenty D-schools (566 pupils), and twenty-three EA-schools (835 pupils).

Each subject took two subtraction tests, one in the morning and one in the afternoon of the same day. For the junior subjects each test contained 28 examples involving a total of 112 separate operations; for the senior subjects, 28 examples also, involving a total of 140 operations. The children were told to "proceed at their own rate without hurry or undue delay." On finishing their tests the children copied from the blackboard the number of quarter-minutes during which they had been at work. In this way two measures were secured for each child on the combined tests, a measure of accuracy (expressed in terms of errors) and a measure of speed or rate of work.

*Experimental Results.*—To test the relative merits of C, D, and EA, Murray determined the mean differences between the experimental groups in errors and in rate of work; then calculated the reliability of the differences in means and the Critical Ratios. For his accuracy data he used not one statistical formula, as is customary, but four in the case of the junior subjects and two in the case of the senior subjects. For his rate data he used, for both groups, a single formula, namely the "short method,"

$$\sigma_{M_D - M_C} = \sqrt{\sigma_{M_D}^2 + \sigma_{M_C}^2}.$$

The Critical Ratios for accuracy are as follows:

|                           |                                                                                                  |
|---------------------------|--------------------------------------------------------------------------------------------------|
| for the younger subjects: | C vs. D, 5.09, 5.66, 5.66, and 3.17<br>EA vs. C, .868, 2.24, 1.09<br>EA vs. D, 7.29, 5.76, 1.84. |
| for the senior subjects:  | C vs. D, 4.57, 5.68<br>EA vs. C, 6.21, 3.25<br>EA vs. D, 8.45, 5.53.                             |

The Critical Ratios for rate are:

|                           |                                                    |
|---------------------------|----------------------------------------------------|
| for the younger subjects: | C vs. D, 3.04<br>EA vs. C, 2.15<br>EA vs. D, 1.34. |
| for the senior subjects:  | C vs. D, 3.44<br>EA vs. C, 2.39<br>EA vs. D, 1.73. |

In both the junior and senior groups, in both accuracy and rate, and in every comparison, D was found to be inferior whether compared with C or with EA, and in amounts large enough, in most cases, to be highly reliable. It is not surprising therefore that the Committee on Primary School Subjects which sponsored the study recommended (18, p. 67):

From the above "Summary of Statistical Results" it is evident that in respect to *accuracy* the results are so decisively in favour of the complementary and equal-additions methods that the Committee feels justified in recommending the complete abandonment of the decomposition method of teaching subtraction. There is a slight difference in favour of the equal-additions method, and in view of the advantages of the adoption of a uniform method throughout the schools of Scotland the Committee recommends the adoption of the equal-additions method.

In respect to *speed* the decomposition method is likewise decidedly inferior to both the other methods, and this reinforces the recommendation of its total rejection. The complementary method is somewhat superior to the equal-additions method in speed, but this is not sufficient to outweigh the superiority of the equal-additions method in accuracy.

*Criticism of the Murray Experiment.*—Unlike the Johnson study and many others, the Murray study cannot be criticized in respect to the subjects used. Murray recognized that subtraction procedures must be evaluated for difficulty at the time of learning as well as for proficiency in later grades. On this account, his data for the junior subjects are especially important, for they help to fill in a gap in our knowledge. By contrast, his data for his senior subjects are less

significant, for they but corroborate facts which have been pretty well established in several other investigations.

(a) ACCURACY DATA. In the junior group the means of errors were: D, 20.9 ( $N = 464$ ); C, 13.9 ( $N = 316$ ); EA, 12.9 ( $N = 842$ ). Since two tests were used, each involving 112 operations, these means of errors are to be compared with the *opportunity* for errors, namely, 224. Expressed as per cents of *accuracy*, the means quoted above become, respectively, 90.7, 93.8, and 94.2; and the differences do not appear so striking.

For the groups matched in accordance with two other formulas the means of errors were: D, 15.4; C, 11.2; and EA, 9.7; and the corresponding per cents of accuracy are: 93.2, 95.0, and 95.2. For the matched groups used with a fourth formula, the means of errors were: D, 22.0; C, 16.9; and EA, 13.2, with corresponding per cents of accuracy: 90.2, 92.5, and 94.1.

True, when measures of accuracy are substituted for measures of error, D still appears to be the poorest procedure and EA the best, with C better than D and poorer than EA to about the same extent. But the point is that the differences are not very impressive; they amount, at the most, to four operations (separate subtractions) per hundred—and this at the time of initial instruction.

What was the situation two years later? The means of errors for the senior subjects for the experimental groups as wholes are reported as: D, 15.9 ( $N = 566$ ); C, 10.7 ( $N = 274$ ), and EA, 8.9 ( $N = 835$ ). On the two tests taken by these children 280 errors were possible. The per cents of accuracy are therefore: D, 92.9; C, 95.2; and EA, 95.6. For the matched pairs used with Murray's third formula the actual means are not given, but the differences in means are: D vs. C, 4.3; C vs. EA, 1.7; D vs. EA, 2.7. Converted into per cents of accuracy these differences are, respectively, 1.9, 0.8, and 1.2; and they are similar in their relationships to the differences for the groups as wholes: 2.3, 0.8, and 2.7. Viewed in this way, differences in the relative accuracy of the three procedures two years after initial instruction are not very significant *educationally*, however reliable from the standpoint of statistics.

The use of error measures serves to "inflate" the amount of difference among the procedures. It does so because the measures are treated absolutely, without regard for the bases with which they should be compared. It should be obvious that a mean of twenty errors with a possibility of fifty errors is quite different in its significance

from a mean of twenty with a possibility of 200 errors, or 500, or 2000. On the other hand, the use of accuracy measures in the comparisons serves to decrease the amount of difference, and for the same reason. Certainly measures of the latter sort are as valid as are those of error. If so, the case for C and EA is greatly weakened, so much so that superiority of D according to other criteria—if such superiority could be established—might easily offset the small advantage of EA and C in accuracy.

(b) EFFICIENCY AS THE SOLE CRITERION. Murray took measures of rate and accuracy of work as his sole criteria to determine the comparative worth of D, C, and EA. The objection raised to the criterion of efficiency in the case of the Johnson study (see pp. 9-10 above) therefore applies to the Murray study as well.

(c) THE TECHNIQUE OF DIFFERENTIAL TESTING. The adequacy of differential testing as a means of deciding the controversy regarding subtraction procedures has been discussed above (pp. 10-11) in connection with the Johnson study. Lack of information respecting methods of teaching employed with the experimental sections greatly diminishes the assured interpretation of the data obtained.

#### OBSERVATIONS ON OTHER EXPERIMENTATION

*Advantages of EA.*—As has been stated, experimentation has rather consistently demonstrated the superiority of EA over D in accuracy and speed. McClelland (15) found that after long practice children aged  $12\frac{1}{2}$  to  $13\frac{1}{2}$  could subtract a little faster by D than by EA, but that even so EA was more “adaptable” (when, without recasting the examples, the “minuend” figures were to be subtracted from the “subtrahend” figures). Winch (29) reported that girls, aged 12, previously taught by D, could, after a short period of instruction on EA, do better than did a paired group who continued with D, but that the change in procedure at this late time seemed hardly justifiable.

*Advantages of D.*—There are a few instances, besides the McClelland study just cited, in which D has been found superior to EA. Taylor (26) and Beatty (2) discovered that, though their subjects had been taught the Austrian Method (which includes features of EA), they tended to abandon it in favor of D. And Wilson (28) through a questionnaire established the fact that in this country two and a half times as many teachers teach D as teach EA. From these data he argues that D should probably be taught, since teachers know it better and should therefore be able to teach it more effectively.

*Beginners as Subjects.*—It has been customary to obtain data on subtraction procedures from subjects considerably removed in time from initial instruction. Thus, Johnson (10) as late as 1938 included in some of his comparisons scores made by forty-three adults.

Only in a few investigations besides that of Murray (18) reviewed just above have experimental subjects been at or near the point of initial instruction. Ballard (1) found EA superior at all ages, 8 to 14; that is, clearly superior in accuracy and less so in rate. Winch's experiment (29) with children aged  $8\frac{1}{2}$  shows some advantage for EA, but he himself discounts the significance of his results, for his subjects knew so little about subtraction that they scarcely profited from instruction. Roantree's data (22) for Grades 2, 3, and 4 are none too reliable. In Grade 2B he found superiority for the Austrian Method (and so, for some features of EA); in Grade 3, none. Osburn's tests (19) were handled better by EA-children than by D-children in Grade 4 in accuracy (he tested only for accuracy, and in Grades 4-6), though, after all, these subjects were tested a year after starting on their procedures. Larcombe (12), in a study cited by Murray (18, p. 25) but not available to the writers, found in favor of EA with 9000 children aged 9, to whom he gave the Ballard tests.

*Experimental Techniques.*—Differential testing has been by all odds the most popular technique employed, having been used in twelve studies: 1, 2, 5, 7, 9, 10, 12, 15, 18, 19, 22, and 26. The controlled-group technique has been used in only one investigation (29; the report cited actually contains data from two separate studies).

#### THE NEED FOR A BROAD EVALUATIVE BASE

Writing in 1925, Knight, Ruch, and Lutes (11) protested the tendency at that time to accept as finally decided the relative merits of EA and D. In that year only seven pertinent experimental reports had been published. While five of them (all except 2 and 26) contained evidence in favor of EA, this evidence, in the opinion of Knight, *et al.*, was wholly inconclusive. They readily admitted their bias for D;<sup>6</sup> yet they undertook to hold this bias in check while they argued, in what is still a most illuminating article, that the issues had not been settled and that they could not be settled until trustworthy

<sup>6</sup> Long before reaching this point in the text, the critical reader will have detected the writers' prejudice in favor of D. In this respect they have tried to take the position of unbiased analysis so well described and exemplified by Knight and his colleagues in the reference cited above.

data were at hand to permit much more comprehensive evaluation than had as yet prevailed.

As bases for this comprehensive evaluation these writers proposed thirteen criteria, some of which, they point out, apply rather more to the controversy between additive and subtractive subtraction than to that between D and EA. Even so, it is worth while to quote the thirteen criteria in full.

1. Which method allows the most helpful transfer from previously learned skills?
2. Which method avoids the most negative transfer or interference from skills previously learned, or to be learned subsequently?
3. Which method uses the simplest process?
4. Which method carries the most useful contribution to other aspects of the total subject?
5. Which involves the mastery of the fewest facts?
6. Which can be carried furthest into the more complicated skills with reasonable ease?
7. Which can be most adequately rationalized?
8. Which admits of the most effective drill devices?
9. Which contributes best to thinking in problem solving situations?
10. Which admits of better checks?
11. Which can be best illustrated by object teaching at the beginning of instruction?
12. Which seems to be the most natural?
13. Which is defended by experimental evidence?

As the advocates of these criteria themselves state, there is considerable overlapping among the criteria. Thus, criterion 1 as well as criteria 2 and 12 (both, in part), relate to transfer from prior learning; criteria 2 (in part), 4, 6, and 9 have to do with transfer or extension and application to later learning; and criteria 7 and 11 deal with the question of intelligibility of process at the time of learning. Nevertheless, telescoped and reduced in number as much as may be, the criteria still call attention to aspects of learning and of use which had been disregarded or but lightly regarded in the research prior to 1925. Nor, in this respect, has the situation changed materially in the research of the past twenty years. As the brief review above reveals, research on the comparative merits of D and EA still makes the issue turn on a very narrow base.

Consider, for example, criterion 6. The truth is that we have practically no data at all on the usefulness of D and EA with anything

except whole numbers. Yet, obviously we have to borrow<sup>7</sup> when we subtract with common fractions and with denominate numbers. One of the present writers has made it a practice for some years to ascertain the subtraction procedures employed by his graduate students in dealing with whole numbers. (Most of these students are classroom teachers.) About a fifth of the students have said that they use EA; yet of this number not more than one out of six uses EA with common fractions and denominate numbers. If Knight, *et al.*, are correct in their view—and the writers think that they are—it would seem preferable to teach the one procedure which is most “naturally” applied to all kinds of subtraction situation; and this would seem to be D rather than EA.

The present study, unfortunately, has nothing to contribute with respect to the issue just discussed: it does not deal with extensions of subtraction procedures beyond their use with whole numbers, and these numbers limited to four places, nor does it deal with some of Knight's other criteria. Nevertheless, it does make use of a wider evaluative basis than has been common, for it presents data on relative ease of learning and of teaching the two procedures, and these data go beyond the customary measures of rate and accuracy of work. To this extent, then, it approaches the definitive kind of study which was envisaged by Knight, *et al.*, two decades ago.

#### THE PROBLEMS OF THE PRESENT RESEARCH

The foregoing criticisms of previous research foreshadow the major characteristics of the present study. This study is an investigation of the relative merits of D and of EA as subtraction procedures. The experimental subjects were third-grade children who were having their first experiences with borrowing in subtraction. The experimental technique of differential testing, for reasons already stated, was rejected in favor of the controlled-group technique, in order to secure as accurate information as possible concerning possible relations between methods of teaching and success with the two subtraction procedures. Two widely dissimilar instructional approaches were set up, the one designed to teach the two procedures rationally or meaningfully, the other, mechanically. (The two approaches are fully explained in Chapter II.) Evaluative criteria

<sup>7</sup> From this point on the word *carry*, which is appropriate as a description when one subtracts in certain ways in examples like  $472 - 266$ , will be omitted in favor of the single word *borrow*. While this practice does violence to a real difference in procedure, the gain in simplicity of expression serves to offset the loss in nicety of terms.

include, of course, measures (a) of rate and (b) of accuracy, but they include also measures (c) of the smoothness of thought processes or work habits in subtraction, (d) of understanding of procedures, (e) of transfer to untaught types of subtraction, and, to some extent, (f) of retention or later gains or losses in efficiency and understanding.

The details of experimentation by which data were secured for these various criteria will be explained in Chapter II. At this point it may be wise to discuss one aspect of the experiment, namely, that relating to rationalization and understanding.

### *The Issue of Meaning*

In the research to date on the problem of D vs. EA, little attention (almost none) has been given to the question of whether children understand the procedures by which they subtract. One reason is the genuine doubt on the part of some students of arithmetic instruction that children need to understand what they learn. For them it is enough if children develop high degrees of skill, no matter how devoid of meaning. Ballard (1, p. 12) stated their position thirty years ago: "... for pure practical efficacy it is better that the rationale should not at the actual time of working be thought of at all. The pupil should confine his efforts to a rigid application of the rule." The "intelligence" afforded by rationalization is only a "temporary illumination." To take time to engender understanding is wasteful, and the effects may be merely perplexity on the part of learners. If rationalization is provided at all, it should come some time (years) after initial instruction. The best methodology, therefore, is to give children at the outset the procedure we want them to have, and then to furnish an abundance of practice. Acceptance of this point of view accounts in large measure for the use of efficiency (rate and accuracy) as the sole criterion for deciding among the different subtraction procedures.

During the last twenty years or so, students of arithmetic in increasing numbers have been taking a different position in the matter of meaning or understanding. Admittedly, they can point to little research in arithmetic to support them (not more than a score or two of studies). For the most part they have had to rely upon arguments, two of which are mentioned here. One argument is negative: mechanical instruction in arithmetic, which has dominated American schools for the last generation and longer, has not produced the kind

of quantitative competence which is demanded by modern society. (This is not the place to assemble the evidence for this statement, but there is plenty of it.) In other words, the argument goes, since mechanical learning gets nowhere, a change is called for, and the logical change (or what is apparently the logical change) is a shift to meaningful learning.

The second argument is based upon educational research (other than in arithmetic) and, much more importantly, upon psychological research in which it has been unmistakably shown that meaning facilitates learning. For example, Lyon, reporting in 1914, stated that his subjects required, on the average, 93 minutes to learn 200 nonsense syllables, 85 minutes to learn 200 digits, 24 minutes to learn 200 words of prose, and 10 minutes to learn 200 words of poetry.<sup>8</sup> And Reed found that it took his subjects, college students, an average of 111.25 seconds to learn nine lines of simple prose narrative, while it took them an average of 261.25 seconds to learn the same amount of material from Hume's *The Origin of Ideas* (which assumedly was less meaningful to them).<sup>9</sup> In a retention test his subjects recalled an average of 49 ideas of the total of 67 in the simple narrative as compared with an average of 11.5 of the 67 ideas in a passage of the same length from Hume.

So consistent is the evidence in psychological research respecting the contribution of meaning to learning that McGeoch in his scholarly summary of data on human learning states: "It is probable, on the basis of available data, that there is a very high positive correlation, and perhaps a perfect one when other things are equal, between meaning and rate of learning."<sup>10</sup> And again:

When the meaning of a material is not easily available to a learner, he may accelerate his rate of learning by a search for meanings, by the imposition of rhythm and pattern, by new groupings of the items, by noting spatial relations, and by other devices whereby he may make the material more meaningful and thus assimilate it more readily into his already existent pattern of responses. . . . The conclusion that there is a high positive correlation between meaningfulness of material and rate of learning holds under a very wide range of conditions.<sup>11</sup>

<sup>8</sup> D. O. Lyon, "The Relation of Length of Material to Time Taken for Learning," *Journal of Educational Psychology*, V (1914), 1-9.

<sup>9</sup> H. B. Reed, "Meaning as a Factor in Learning," *Journal of Educational Psychology*, XXIX (1938), 419-430.

<sup>10</sup> John A. McGeoch, *The Psychology of Human Learning* (New York: Longmans, Green and Co., 1945), p. 159.

<sup>11</sup> *Ibid.*, p. 167.

In view of the standing accorded by psychologists to meaningfulness as a factor in learning, it seems strange that any students of arithmetic remain unready to give it a prominent place in their subject. Perhaps the explanation lies in an oversimplification of arithmetic as learning material, an oversimplification which reduces arithmetic to the level of a vast number of isolated elements which must be mastered by memorization, like so many nonsense syllables. Yet, as McGeoch has pointed out, even in mastering nonsense syllables learners endeavor to impose meaningful system upon their relatively meaningless learning tasks.

Be all this as it may, in the present investigation arrangements were made to measure the effects of meaningfulness of learning task. In the first place, as already explained, the two subtraction procedures, S and EA, were taught both meaningfully (through rationalization) and mechanically. (Thus, there were four experimental sections, DM, Decomposition Mechanical; DR, Decomposition Rational; EAM, Equal-Additions Mechanical; and EAR, Equal-Additions Rational.) In the second place, there were known to be distinct differences among the co-operating schools in the extent to which arithmetic had been meaningfully taught in the first two grades. It was therefore possible to note the relative success of teaching borrowing meaningfully and mechanically to children with considerably unlike backgrounds with respect to understanding. In the third place, measures of the effect of meaning were secured with regard to the comparative ability of M-subjects and of R-subjects to transfer their procedures to untaught types of subtraction examples. In the fourth place, the extent to which meanings were actually attained in the subtraction procedures—whether these meanings had been taught or not—was studied directly by interviewing all subjects once and most of them twice. In the individual interviews the subjects were asked to explain what they did (thought) when they borrowed.<sup>12</sup>

As one means of instilling meaningfulness into the procedures taught by R (both the DR- and the EAR-subjects), rather elaborate "crutches" were taught. Since considerable objection to such learn-

<sup>12</sup> The reader is again reminded that the single term *borrowing* is used to refer to *whatever* the child or adult does when he alters the minuend or both the minuend and subtrahend in such examples as  $450 - 346$ . Obviously, in EA one does not "borrow" at all; instead, he adds 10 ones to the minuend and a ten to the subtrahend. The use of the single term is justified only by interest in simplicity of expression.

ing aids<sup>13</sup> still exists, it became a subsidiary problem in the present research to note the way in which these devices persisted or disappeared in the behavior of the children who learned them.

#### SUMMARY

This chapter is intended to explain the problem of the present investigation and to show the need for continued research on this problem. Practically all research on the relative merits of D and EA (a) has employed as subjects school children and even adults who were long distant in time from the period of learning. Consequently, it (b) has furnished little information on the ease or difficulty of learning the two subtraction procedures. Moreover, this previous research (c) has restricted evaluation to a single criterion, efficiency (as measured by rate and accuracy of work), and (d) has not controlled variations in methods of instruction as factors in learning. Because of the particular psychology of learning and because of the methods of teaching arithmetic which prevailed during the period of this research, it is probably safe to assume that the experimental subjects (save some of Winch's, 29) were taught their subtraction procedures purely as "mechanical skills," with little or no rationalization.

In the present study the experimental subjects were school children who were just beginning to "borrow." Very unlike methods of teaching both procedures were prescribed, so that half the children were taught meaningfully, and half mechanically, and the evaluative criteria were extended beyond measures of rate and accuracy to include: evidence of understanding, ability to transfer, smoothness of work habits, and the testimony of teachers respecting difficulties of teaching the two procedures by the different methods.

At the beginning of this chapter the statement was made that the investigation has both educational and psychological significance—educational, because it is imperative that we learn how to teach subtraction better; psychological, because after all the experiment is on learning and because it deals with meaningfulness of the learning task as a factor in learning. The educational and the psychological aspects of the study are inextricably bound up. It seems hopeless—and useless—to try to separate them. Hence, no attempt will be made to do so in this report.

<sup>13</sup> One of the crutches, that used in connection with D, has previously been investigated (4). More recently Wilburn (27) has brought new evidence to support the use of the D-crutch.

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## CHAPTER II

### PLAN OF THE EXPERIMENT

#### THE SUBJECTS

The subjects of the experiment were approximately 1400 third-grade children enrolled in forty-one classes in the schools of Burlington, Charlotte, Greensboro, and Winston-Salem, North Carolina. With the exception of repeaters, who were eliminated in the statistical treatment of the results, none had as yet had any experience in borrowing. They had learned to carry in addition, and at the start of the Pre-experimental Period were at work on such subtraction examples as those at the right, herein, for purposes of brevity, to be called nonborrowing examples. (It has already been explained that the term "borrowing" includes both the alteration of the minuend in D and the alteration of the minuend and the subtrahend in EA.)

|            |            |             |             |
|------------|------------|-------------|-------------|
| 79         | 68         | 708         | 483         |
| <u>- 5</u> | <u>-34</u> | <u>-507</u> | <u>-450</u> |

#### EXPERIMENTAL CENTERS: A, B, AND C

The arithmetic programs for Grades 1 and 2 differed considerably among the four school systems, a fact which was known to the senior author by reason of repeated classroom visits over a period of years, and a fact further substantiated by conferences with various school officials and members of the teaching staff at the start of the experiment proper.<sup>1</sup>

In one of the cities considerable systematic work in arithmetic, with emphasis upon meaningful experiences, is done in Grades 1 and 2. Individual workbooks, designed to develop insights into the various number tasks, are used in the schools of this city beginning with

<sup>1</sup> It was planned to get objective evidence of these differences by means of a specially constructed test. This test consisted of seventy-nine Yes-No questions which measured meanings in connection with: the composition of numbers in terms of place value, the process of addition, carrying in addition, the process of subtraction (nonborrowing), and the like. Each teacher, in order to obviate reading difficulties for the pupils, read the test items slowly, and the children were expected to record each answer by encircling either Yes or No. The test results had to be discarded. Quite unintentionally the teachers, by stressing key words in the items, furnished clues to the correct answers. Were the investigation to be repeated, the writers would give this test again themselves, for they are convinced that, properly administered, it would yield valid data of real importance.

the second half of Grade 1. These children entered the experiment, it was thought, with an arithmetical background quite unlike that enjoyed by the children in the other three cities. Accordingly, the twelve third-grade classes in this city are regarded as a special group of subjects; the city is designated as Center A; and the experimental results obtained there are treated separately (Chapter III).

In another of the four school systems there is considerable *talk* about meaningful arithmetic in the first two grades. Actually there is some, but not much. What there is, is spread unevenly throughout the system, only an occasional primary-grade teacher working seriously for understanding. On the other hand, *as a whole* the arithmetic program in Grades 1 and 2 is heavy, and children enter Grade 3 with more training in simple computation than they do in the other three cities. The twelve classrooms drawn from this city are grouped together to make Center B (Chapter V).

Center C (Chapter IV) consists of seventeen classes, nine from one city system, eight from another. In these cities there is little arithmetic in Grades 1 and 2, and that little is abstract, being taught with a minimum of attention to meaningful mathematical relationships. In order to get their pupils ready for this experiment the teachers in this center had to resort to strenuous measures in the Pre-experimental Period to make good their arithmetical deficiencies.

The study, then, comprises three independent but parallel experiments, for in all centers the same tests, experimental schedules, and instructional directions were employed. Because of the marked differences in pre-third-grade arithmetic it seemed unwise to throw together the data for the forty-one classes. To have done so would have been to obscure any possible effects of the unlike instructional programs. On the other hand, the plan to keep the three groups of subjects separate provided a way to isolate meaningfulness as a factor in learning. In other words, it made it possible to test the extent to which success in meaningful and in mechanical teaching is conditioned by the nature of prior learning.

#### EXPERIMENTAL SECTIONS: DM, DR, EAM, EAR

The children in each of the three centers were divided in the same manner so as to yield four experimental sections. In Centers A and B, each experimental group contained three classes; in Center C, four or five classes. Half of the classes in each center learned to borrow by D; half, by EA. Each half was divided again, so that

one part learned their subtraction procedure meaningfully (R, rationally); the other part, mechanically (M). Thus, there eventuated the four experimental sections: DM, DR, EAM, EAR.

Within each center the experimental sections were set up so as to be as nearly as possible equal in ability. The original allocation of classes was done at a meeting attended by the co-operating teachers and by the city supervisor and/or superintendent. If a class known to be of superior ability was assigned to the EAR section, for example, another class, known to be below average by about the same amount, was put into the same section, so as to balance with other sections. Admittedly, the process was subjective, and some errors were made; but the step was worth taking, for it yielded sections that were roughly comparable at the start and greatly reduced the difficulty of the more careful equating of groups which had to be done later before the experimental results could be treated statistically.

#### ASSIGNMENT OF TEACHERS

The senior author first secured the consent of the city superintendent and supervisor to allow their teachers to participate in the experiment. He then met with all the third-grade teachers in the city system, explained the purpose of the experiment, and invited co-operation. Teachers who wished to be included in the experiment, knowing pretty well the demands which would be made of them, were asked to give their names to the supervisor or superintendent within the next week.

The senior author then met with the teachers who had volunteered. At this time he demonstrated the two subtraction procedures (D and EA) and explained in a general way the differences in teaching methods (R and M) to be used with both procedures. The teachers then indicated which one of the four experimental types of class they preferred to teach. With but few exceptions it was possible to assign teachers according to their wishes; and the few exceptions were in every instance given their second choices. There were always three to five teachers who were willing to be assigned to any experimental group, a welcome circumstance, since it was then possible to fill up the four sections and to insure rough comparability of pupil ability among them.

At this second meeting one teacher or the city supervisor was selected to serve as "co-ordinator" for each center, to receive materials and communications from the experimenters, to return test

papers and other materials, and to maintain uniform practices as well as planned differences among the experimental sections. It was the co-ordinator's responsibility to pass on to the experimenters all uncertainties about teaching procedures reported to them by the teachers. It may be said that the co-ordinators functioned conscientiously and intelligently.

#### DIRECTIONS FOR TEACHING

Two weeks before the start of the Experimental Period (see below) all teachers received mimeographed directions to govern their methods of teaching. Four sets of instructions were prepared, one for each experimental section. Copies will be found in Appendix C.

The teachers were requested to study their instructions together with the textbook, to which the daily lessons were keyed. Each day's work was prescribed for the fifteen days of the experimental period, and teachers were told both what to do and what not to do each day. Some leeway was allowed in the schedule. For example, sets of extra practice were supplied, to be used as need required, or as time permitted. Also, teachers of R-sections were advised to proceed slowly at first, taking an extra day or two for the first lessons if they so desired, and shortening later work in ways which were indicated.

To conserve space at this point, a digest of major differences in instruction follows. The critical reader will want to examine the full statements in Appendix C.

#### The R-Sections

For *DR-classes* the *final* verbal pattern, illustrated from the example at the right, was to be: "Eight from 15, 7; 2 from 5, 3."

$$\begin{array}{r} 65 \\ -28 \\ \hline 37 \end{array}$$

For *EAR-classes* the *final* verbal pattern for the same example was to be: "Eight from 15, 7; 3 from 6, 3."

In both types of class this short "direct" verbal pattern was to be approached through a series of graded steps in thinking, starting with those which most obviously revealed or pictured the meaning of the operation in question. These first steps are exemplified below.<sup>2</sup>

<sup>2</sup> Actually, as appears in the full instructions in the Appendix, steps prior to those given below were provided, to be used with children who were not ready for Step 1.

## DR-Classes

## Step 1.

| (1)                                       | (2) | (3)                                                           |
|-------------------------------------------|-----|---------------------------------------------------------------|
| 65 = 6 tens + 5 ones, or 5 tens + 15 ones |     | $\begin{array}{r} 56\ 15 \\ -2\ 8 \\ \hline 3\ 7 \end{array}$ |

The item marked (3) illustrates the crutch to which reference has already been made in earlier pages.

## Step 2.

Write the examples at the right on the blackboard. Have the pupils copy them, *using the crutch*. Call it a "number picture."

|                                                    |                                                    |                                                    |                                                    |
|----------------------------------------------------|----------------------------------------------------|----------------------------------------------------|----------------------------------------------------|
| $\begin{array}{r} 52 \\ -36 \\ \hline \end{array}$ | $\begin{array}{r} 40 \\ -31 \\ \hline \end{array}$ | $\begin{array}{r} 35 \\ -16 \\ \hline \end{array}$ | $\begin{array}{r} 54 \\ -38 \\ \hline \end{array}$ |
|----------------------------------------------------|----------------------------------------------------|----------------------------------------------------|----------------------------------------------------|

Develop the following verbal pattern:

"I cannot take 6 from 2 [first example above]; so I borrow a ten from the 5 tens.

"I cross out the 5 and write a little '4' to show that I borrowed a ten.

"I write a little '1' in front of the 2 to show that I now have 12 instead of 2.

"Then I subtract: 'Six from 12, 6,' and I write 6 in the ones' place; 'Three from 4, 1,' and I write 1 in the tens' place."

Have the children "draw the number picture" to show what they do when they subtract. In other words, insist that the crutch be used in every example with borrowing.

## Step 3.

| (1)                                                              | (2)                                                                    | (3)                                                              |
|------------------------------------------------------------------|------------------------------------------------------------------------|------------------------------------------------------------------|
| $\begin{array}{r} 56\ 10 \\ -\quad 7 \\ \hline 5\ 3 \end{array}$ | $\begin{array}{r} 3\ 34\ 16 \\ -2\ 3\ 9 \\ \hline 1\ 0\ 7 \end{array}$ | $\begin{array}{r} 4\ 23\ 15 \\ -4\ 2\ 8 \\ \hline 7 \end{array}$ |

For type example (1), the children are to say merely, "Seven from 60, 53"; for type (2), "Nine from 16, 7; 3 from 3, 0; 2 from 3, 1"; for type (3), "Eight from 15, 7; 2 from 2, 0; 4 from 4, 0." This shorter verbal pattern, once it is introduced, is to be used consistently with the crutch, as shown above.

## EAR-Classes

(1) *New mathematical principle.* The principle that the difference between two numbers is not changed if the same number is

added to both terms is to be developed by using such series of relationships as A and B at the right. For this purpose employ (a) actual objects, (b) drawings of numbers, (c) simple, easily imagined verbal problems, and (d) series of abstract examples as illustrated.

| A             | B             |
|---------------|---------------|
| $4 - 1 = 3$   | $9 - 7 = 2$   |
| $14 - 11 = 3$ | $19 - 17 = 2$ |
| $24 - 21 = 3$ | $29 - 27 = 2$ |
| $34 - 31 = 3$ | $39 - 37 = 2$ |

(2) Application of the principle of borrowing in subtraction.

*Step 1.*

| (1)                                                  | (2)                                                                                                                                                                                                 | (3)                                                   |
|------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------|
| $\begin{array}{r} 65 \\ -28 \\ \hline ? \end{array}$ | $\begin{array}{l} 65 = 6 \text{ tens} + 5 \text{ ones, or } 6 \text{ tens} + 15 \text{ ones} \\ -28 = 2 \text{ tens} + 8 \text{ ones, or } 3 \text{ tens} + 8 \text{ ones} \\ \hline ? \end{array}$ | $\begin{array}{r} 65 \\ -28 \\ \hline 37 \end{array}$ |

*Step 2.*

This step is intended to give the children a way to think about borrowing. For the type example,  $65 - 28$ , they are to think:

"I can't take 8 from 5; so I add a ten to 5 and get 15.

"Now I can subtract the ones: 'Eight from 15, 7'; and I write 7 in the ones' place.

"Since I added a ten to the ones' figure in the top number, I must add a ten to the tens' figure in the bottom number, and 2 becomes 3.

"Now I can subtract the tens—'Three from 6, 3'; and I write 3 in the tens' place."

Insist that the crutch, (3) above, appear in every solution where there is borrowing.

*Step 3.*

Type examples (1), (2), and (3) are rewritten below with the crutch. When the pupils take Step 3, they are to continue to use the crutch, but they will say: for (1), "Seven from 60, 53"; for (2), "Nine from 16, 7; 4 from 4, 0; 2 from 3, 1"; for (3) "Eight from 15, 7; 3 from 3, 0; 4 from 4, 0."

| (1)                                                  | (2)                                                     | (3)                                                   |
|------------------------------------------------------|---------------------------------------------------------|-------------------------------------------------------|
| $\begin{array}{r} 60 \\ -7 \\ \hline 53 \end{array}$ | $\begin{array}{r} 16 \\ -249 \\ \hline 107 \end{array}$ | $\begin{array}{r} 15 \\ -428 \\ \hline 7 \end{array}$ |

### The M-Sections

DM- and EAM-pupils were taught to borrow in a purely mechanical, rote fashion. At no time was the slightest hint offered concerning the rationale of the procedures. Questions from pupils

which probed into such matters were dismissed outright or with "explanations" which explained nothing.

Both DM- and EAM-classes were given only the verbal pattern they were to use. The procedure sequence for both experimental sections followed the suggestions contained in the text used in all schools.<sup>3</sup>

### *DM-Classes*

The first verbal pattern for the type example is to be:

"I can't take 8 from 5; so I think of 5 as 15.

"Then I think, 'Eight from 15 is 7,' and I write 7.

"Since I thought of 5 as 15, I must think of 6 as 5.

"Then I think, 'Two from 5 is 3,' and I write 3."

Next, have the children shorten the verbal pattern to "Eight from 15, 7; 2 from 5, 3."

|     |
|-----|
| 65  |
| -28 |
| 37  |

### *EAM-Classes*

For the type example above, the following verbal pattern is to be used:

"I can't take 8 from 5; so I think of 5 as 15.

"Then I think, 'Eight from 15 is 7,' and I write 7.

"Since I thought of 5 as 15, I must think of 2 as 3.

"Then I think, 'Three from 6 is 3,' and I write 3."

Next, have the children shorten the verbal pattern to: "Eight from 15, 7; 3 from 6, 3."

## THE EXPERIMENTAL SCHEDULE

Three periods of time need to be identified within the course of the experiment as a whole, and the activities assigned to each need to be described.

### *The Pre-experimental Period*

There is no exact way to date the start of the Pre-experimental Period; but in each center its end, varying by a week from center to center but the same for all schools within a center, was marked by the administration of tests prior to the beginning of instruction in borrowing. During this first period the teachers were (a) to insure mastery of the simple subtraction facts on the part of their pupils, (b) to teach subtraction without borrowing with two- and with three-place numbers, and (c) to teach addition, with and without

<sup>3</sup> Fletcher Durrell, Harry O. Gillet, and Thomas J. Durrell, *The New Day Arithmetics* (New York: Charles E. Merrill Co., 1937).

carrying, with numbers of the same orders. They were also (d) to try to make clear the nature of subtraction as a process. And, finally, in order to prepare themselves for their teaching tasks in the experiment, they were (e) to familiarize themselves thoroughly with the subtraction procedure they would teach (D or EA), so that they could use it naturally and would use it invariably during the experiment; and they were (f) to study the instructional directions furnished them, asking their co-ordinator for interpretations wherever the directions were not explicit or were not clear.

Variations in the length of the Pre-experimental Period, and so in the starting time of the experiment proper, arose chiefly in connection with (a) above: largely because of differences in the amount and in the nature of pre-third-grade instruction the children in the three centers had not attained equal command over the subtraction facts.

At the conclusion of the Pre-experimental Period all children took an intelligence test and two tests in subtraction: one on the simple subtraction facts, the other on nonborrowing subtraction (with examples in addition interspersed irregularly throughout).

### *The Experimental Period*

In all centers the Experimental Period lasted for fifteen days, or three school weeks, regardless of the calendar date when instruction on the borrowing procedures was started. During this time the teachers taught according to the directions which have been described.

Both DR- and EAR-sections were taught crutches (see pp. 29-30 above) and were required to use their crutches in all seatwork. To check their use of crutches, teachers of these sections were asked to collect and to send to Duke University two sets of assigned seatwork, those for pages 87 and 98 of the textbook.

At the end of the Experimental Period all pupils were given computation tests in subtraction, and all were interviewed individually. In addition to measures of efficiency and of transfer (see pp. 39 and 40 below), the computation tests also supplied evidence on the use of the crutch.

### *The Postexperimental Period*

The Postexperimental Period followed the Experimental Period as a matter of course. During the six weeks of this period no additional instruction was given on borrowing, and practice on the pro-

cedures was supposed to be limited to the small amount provided by the textbook for the purpose of maintenance.

About two weeks after the close of the Experimental Period a test was given to DR- and EAR-subjects, solely to determine the extent to which the crutches were then in use. (This test therefore did give these classes one practice period which the DM- and EAM-classes did not have.)

At the end of the Postexperimental Period a test was administered to all sections to measure the retention of skills acquired during the Experimental Period, and a second interview was held with all children except those in Center B. All interview and testing materials are reproduced in Appendix B.

#### TESTS USED AND MEASURES SECURED

##### *For Determining Initial Abilities*

(1) The experimental sections in all centers were finally equated in terms of accuracy scores on a pretest (Appendix B, p. 178). This test comprises forty-nine examples, eighteen in addition and thirty-one in subtraction, in random order, all examples representing types of computation with two- and three-place numbers which were supposed to have been mastered during the Pre-experimental Period. Instructions were: "Don't hurry so fast that you will make mistakes." There were no set time limits; instead, the children were allowed to work until all except the two to four or five least capable children had finished. Accuracy scores equaled the number of correct answers. Rate scores were secured by having children encircle the example on which they were busy at various times during fifteen minutes. Reliability of the test, as calculated by the split-halves method and the Spearman-Brown formula, is  $r_{11} = .82$ , sufficiently high for the kinds of group comparison to be made.

(2) About the same time a written test on the one hundred subtraction facts was administered. The directions were the same as for the Pretest, and measures of rate and accuracy were secured in the same way.<sup>4</sup>

<sup>4</sup> In the reports for Centers A, B, and C (chaps. iii-v) the scores on these tests are not presented. In spite of careful and specific directions, it was found that the teachers varied widely in their methods of administering the tests, so much so that comparisons between experimental groups did not seem to be warranted. The loss of this one type of measure for matching groups is of course regrettable, but it is actually not very serious. In the first place, the number of other bases for matching is unusually large. In the second place, scores on these tests would have revealed knowledge of the combinations *in isolation only*, and would have revealed nothing with respect to *functional mastery*, as in computation.

(3) Measures of "intelligence" were obtained through the use of Kuhlman-Anderson Intelligence Tests. The tests were administered by experienced persons, and they were scored by two individuals in order to assure reliability. These tests, together with CA's obtained from the children (and checked by the teachers), gave measures of IQ and MA.

*For Evaluating Results at the End of the Experimental Period*

(1) The relative success of the experimental teaching was determined largely from scores on the End Test (Appendix B, p. 179). This test has three parts. Part I contains eighteen subtraction examples, fourteen of the borrowing type and four of the nonborrowing type. It was supposed that children in EAR- and DR-sections would use their crutches, as they had done in all seatwork. Part II was like Part I in content, but the R-classes were told to omit the crutches, the purpose being to see whether the quality of their work would suffer. Part III, also containing eighteen examples, introduced borrowing from the hundreds' place, a process no group had encountered before, in order to measure the amount of transfer to new types of example. The point of interest here was: Would the R-sections surpass the M-sections in untaught types of borrowing, and how would the EAR- and the DR-pupils compare? The reliability coefficients of the three parts (split-halves method, with correction by the Spearman-Brown formula) are: Part I,  $r_{11} = .86$ ; Part II,  $r_{11} = .84$ ; Part III,  $r_{11} = .92$ . Rate scores for the different parts of the test were secured as for the Pretest, by having children indicate the point of their work after subtracting for four minutes.

(2) Further data on the relative merits of the different ways of teaching borrowing were obtained in individual interviews, as described below, pages 35-39.

(3) Mention has been made of the fact that the pupils' work on Parts I and II of the End Test was examined to discover the extent to which the crutches were in use. Data on the same point were also obtained at earlier times in the course of the experiment, through an examination of pupils' papers for pages 87 and 98 of the textbook. These pages were assigned on or about the fifth and the fourteenth days of the Experimental Period.

*For Evaluating Changes during the Postexperimental Period (DR, EAR)*

(1) To continue with the history of the crutches: (a) the Post-experimental Test (Appendix B, p. 180), given three weeks after the

close of the experiment proper, afforded another chance to see whether the children, no longer required to use the crutches, were still doing so, or had abandoned them. (b) Still further evidence was secured a month later, when the Retention Test was administered.

(2) The study as a whole ended with the Retention Test (Appendix B, p. 180). Strictly speaking, the test did not measure retention unmixed with practice after the Experimental Period. The amount of this maintenance practice was, however, slight; and the changes from the scores obtained on the End Test are actually not far from being measures of retention.

This test was administered like the other computational tests, without set time limits, but in such a way that both accuracy and rate scores could be had (as well as evidence on use of the crutches). Reliability of the test, which consisted of forty-two subtraction examples, all except ten calling for borrowing, is .90 (split-halves method, Spearman-Brown correction).

(3) The second series of individual interviews was held immediately after the Retention Tests.

#### THE INTERVIEWS

##### *Purpose*

Individual interviews were given twice: once to all experimental subjects at the end of the Experimental Period, and again with the subjects in Centers A and C<sup>5</sup> at the end of the Postexperimental Period. The purpose of the interviews was to broaden the base for evaluating the merits of DM, DR, EAM, and EAR by adding criteria, unavailable through group tests, to the traditional criteria of (a) rate and (b) accuracy of work. The interviews afforded chances to check on (c) the use made of the crutches, (d) smoothness of work habits with the subtraction procedures, and (e) levels of understanding achieved under the differing conditions of experimental teaching.

##### *The Interview as a Research Technique*

The interview has not been overly popular as a research technique in educational and psychological investigations of learning. The reasons are understandable. In the first place, interviewing takes a

<sup>5</sup> It was impossible for a number of practical reasons to arrange for the second interviews in Center B. One of these reasons is that the interviews would have come at a particularly busy time in the school program. Another was that a sufficiently large corps of interviewers could not be assembled.

tremendous amount of time as compared with group testing. (It is estimated that over five hundred hours were spent in interviews in the present study.) In the second place—and this is the reason usually given—the interview does not yield data as “objective” and as “reliable” as are test scores. That is, it does not do so when the interviewing is inexpert, for unbridled subjectivity can ruin the results. There is nothing inherent in the interview as a research technique which automatically forestalls abuses or which corrects abuses once they are made. And when the interviewing is done inexpertly, when it is poorly managed, the data obtained deserve precisely the reception which is usually given them.

But interviewing need not be inexpert. Distinguishable categories can be isolated and defined; the questioning can be standardized without making it inflexible; classifications of subjects' responses can be checked for reliability, and so on. When these precautions are taken, the method of the interview can become respectable in research.

The interview needs to be made “respectable,” and it needs to be used in learning research much more widely than is common. For the interview enables the investigator to penetrate further (than by group testing) into the behavior of his subjects and to note significant aspects of that behavior which otherwise would almost certainly elude him. In the absence of these intimate and comprehensive measures of performance, evaluation must necessarily be incomplete.

#### *Evolution of the Experimental Interview*

The interview as it was finally adopted for experimental use evolved from a long period of trial during which many changes were made. The writers first interviewed a half dozen third-grade children on four or five borrowing examples, taking careful records of (a) the subjects' free reports of procedures and explanations, (b) the questions which seemed to be needed to elicit more complete statements, and (c) the subjects' responses thereto. A tentative blank and tentative categories were worked out and used with several other third-graders. Modifications were unmistakably necessary. For example, (1) the original categories were ambiguous and had to be more specifically defined; (2) the children needed preliminary practice in “thinking out loud”; so two unscored examples were introduced to start the interview series; (3) smoothness of procedure had to be identified as distinct from unequivocal evidence of understanding.

The revised categories, blank, and procedure were tried out again with different children, and a few more changes were made. When a provisionally sound technique seemed to have been perfected, several students (seniors and graduates) were asked to interview two or more children apiece, always under supervision. Their difficulties and criticisms were collected, and the indicated changes were made. The new technique was tried out once more by the same students. This time it seemed to be satisfactory; but it was not wholly so, for the first experimental interviewing uncovered ambiguities at two points and required the addition of two subcategories ("x" and "y," as explained below), in order to insure complete coverage of relevant phases of the children's behavior. Once these changes were made, no others were required.

### *Training of Interviewers*

With some 2500 interviews to be given it was necessary to recruit interviewers to supplement the activities of the writers. The students already referred to (see Acknowledgments for their names, and for the names of others who co-operated) were available and were known to be competent, by reason of their training during the development of the interview technique. Through the kindness of Dr. Noble R. McEwen, ten students majoring in education at Salem College were secured to help in the interviewing in Winston-Salem. Dean James M. Godard and Mrs. Estelle R. Porter of Queens College furnished nine of their advanced students in education for the interviewing in Charlotte. Miss Eugenia Echerd, of Greensboro College, supplied twelve of her education majors to help in the Greensboro system.<sup>6</sup> The interviewing in Burlington was all done from Duke University.

To prepare the co-operating students for interviewing and to maintain the needed control during the actual interviewing, the following steps were taken:

(a) Interview blanks and other supplementary material were sent to Dr. McEwen, Mrs. Porter, and Miss Echerd in sufficient quantity for distribution among the students. All—instructors and students alike—studied these materials carefully before the meetings described in step (b).

(b) One or the other of the writers (Brownell or Moser) spent two days with each group of students. First he explained the experi-

<sup>6</sup> The college students all volunteered for the task and received course credit for what they did.

ment and described the place of the interview therein. He then went through the interview materials with the students, ending with two or more demonstrations with children. Finally, in pairs the students interviewed school children (none of them experimental subjects) under supervision. Their records were compared for agreements and disagreements. This process was continued until it was clear that all students knew how to interview and how to record the interview results.

(c) In each period of experimental interviewing one of the writers or one of the graduate students whose names appear on the cover of this monograph was always present with each group of students, to direct and to supervise their interviewing. Difficult questions of classification and interpretation were settled by the "supervisors" on the spot; or complete statements were entered on the blanks, to be used by the investigators to make the decision at a later time.

#### *Procedure in the Interviewing*

Teachers sent one pupil at a time to an isolated room in the school building where he met his interviewer. After a few minutes of friendly conversation, the interviewer laid before the child a slip of paper containing the following eight subtraction examples:

| (1)         | (2)         | (3)         | (4)         | (5)         | (6)         | (7)         | (8)         |
|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| 945         | 876         | 375         | 797         | 650         | 532         | 893         | 434         |
| <u>-636</u> | <u>-342</u> | <u>-268</u> | <u>-432</u> | <u>-437</u> | <u>-419</u> | <u>-673</u> | <u>-218</u> |

The first example (borrowing), intended to orient the child to the situation and give the interviewer a general idea of the child's performance, was worked silently. The subject then worked example 2 (nonborrowing) "out loud." Thus he became completely familiar with the interview procedure before the test examples, beginning with 3, were presented. No entries were made on the record blank for the first two examples.

Of the six test examples that followed, 3, 5, 6 and 8 called for borrowing, and 4 and 7 did not. The nonborrowing examples were interspersed in order to prevent the children from establishing a pattern which could be used throughout the interview.

Work on examples 3 to 7 was rated while the child reported aloud. For the borrowing examples the child's performance was scored (a) on procedure (D, EA, or neither); (b) on use of the crutch (C) or nonuse thereof (N); and (c) on quality of process: C or N accompanied by the subscripts 1, 2, 3, or 4 to indicate degree of smooth-

ness, the number 1 representing the highest level of performance. If a child forgot that he had borrowed, the letter "y" was entered after the number indicating the smoothness of his work habits, as:  $C_{1y}$  or  $N_{2y}$ . On the nonborrowing examples only quality of process was recorded, the code symbol being N (noncrutch) followed by the numbers 1, 2, 3, or 4 to indicate degree of smoothness. The letter "x" was added to the notation if the child acted as if he had borrowed from tens' or hundreds' place, as  $N_{2x}$ . (Entries of the types  $N_{2y}$  and  $N_{2x}$  were found, upon analysis, to provide no useful information. Hence, they are disregarded in subsequent chapters.)

After the child had worked examples 3 through 7 inclusive, the interviewer directed attention again to the first borrowing example (3) and asked him to explain what he had done. Leading questions which might give cues were avoided, the preferred method being to offer absurd "explanations" which the child would reject. His degree of understanding for the example was recorded as  $U_1$ ,  $U_2$ ,  $U_3$ ,  $U_4$ , or  $U_5$ . Subscript 5 represents utter confusion, whereas subscript 1 means a clear understanding and a lucid explanation. The same procedure was followed for the other two borrowing examples, 5 and 6.

The last example, 8, was used only when the interviewer had up to then been unable to decide whether the child did or did not use a crutch. In such instances the interviewer altered the example himself and asked the child whether he ever used or ever had used that device.

Typically (that is, when example 8 was not used), a total of fourteen measures were obtained: four measures each on the three borrowing examples, and one each on the nonborrowing examples. A sample of the Interview Blank, on which entries were recorded, will be found on page 146 of the Appendix. Pages 181-186 of Appendix B give in detail the instructions furnished the interviewers: Directions and Questions, and Code Letters and Digits.

#### EVIDENCE ON USE OF CRUTCHES

To recapitulate facts which have already been mentioned, evidence on use of the crutches taught the R-sections was obtained at five different times over a period of ten weeks (fifty school days), as follows:

- |                    |                                |
|--------------------|--------------------------------|
| (1) fifth day      | : Practice page 87 of the text |
| (2) fourteenth day | : Practice page 98 of the text |
| (3) sixteenth day  | : End Test                     |
|                    | First Interview                |
| (4) thirtieth day  | : Postexperimental Test        |
| (5) fiftieth day   | : Retention Test               |
|                    | Second Interview               |

The purpose of taking these "soundings" at intervals was to discover whether children taught these learning aids persist in their use or abandon them when they feel that they need them no longer. Fear of long-continued use is given in educational literature as one of the chief reasons for denying these aids to children.

#### TEACHERS' LOGS

In order to employ another evaluative criterion with respect to the four types of experimental instruction, the teachers were asked to keep day-by-day records of their activities and of their observations of significant pupil behavior. The reason for the diaries was explained, as well as the critical importance of reporting intelligently and accurately all classroom events which might reveal peculiar difficulties or peculiar advantages of the methods they were expected to teach.

All except three of the forty-one experimental classes are represented in diaries furnished by teachers at the close of the Postexperimental Period. As would be expected, the logs vary greatly in insight, in detail, and in completeness. The more crucial data for each center will be summarized in Chapter VI.

#### PLAN OF STATISTICAL TREATMENT

The experimental procedure was designed to test the relative effectiveness of four different instructional programs. On the assumption that within each center the experimental sections were initially comparable so far as certain factors are concerned (MA, IQ, rate of work, and accuracy of work in nonborrowing subtraction), a method had to be found which would evaluate differences in final learning status.

Various statistical formulas are available for testing the reliability of differences in means. All the formulas involve certain assumptions with respect to the groups compared—such as whether the experimental groups are made up (a) by pairing subjects, one subject in each group with one in each of the other groups, or (b) by matching

groups less exactly in terms of means and standard deviations, as was done in the present instance.

Complications are introduced in method (a) when more than a single factor is used for equating. In the present study where four such factors were employed, "quartets" of subjects rather than "pairs" would have been required, with consequent serious losses in populations. Furthermore, there is reason to question whether such pairing serves its purpose without at the same time exposing the statistical treatment to new difficulties.<sup>7</sup>

As for method (b), most of the recognized formulas assume matching with respect to a single variable. This method could of course have been used, in which case it would have been necessary to make up a different set of matched sections in each center for each of the four initial factors. The sections so obtained with respect to this one controlled variable would then have had to be compared with reference to each of the learning criteria by which final achievement was assessed. Of these criteria there are sixteen, representing the following six types: rate in borrowing in subtraction, accuracy on the same, rate on a transfer test, accuracy on the same, smoothness of operation, and degree of understanding. If nothing further were done, each relationship (e.g., the relative effectiveness of the instructional programs with respect to rate of work, MA held constant; the same, with IQ held constant; and so on) would stand in isolation; whereas it is certain that the factors MA, IQ, and initial status in the skill under consideration do not so operate in isolation.

For this and other reasons it seemed wiser to combine the four matching factors<sup>8</sup> in the manner suggested by Shen,<sup>9</sup> and to use his formula:

$$t = \frac{x_1 - x_2}{\sqrt{\frac{\sum(x_1 - \bar{x}_1)^2 + \sum(x_2 - \bar{x}_2)^2}{N_1 + N_2 - 3}}} (1 - r_{xy}) \left( \frac{1}{N_1} + \frac{1}{N_2} \right)$$

<sup>7</sup> For example, Shen says: "The adoption of a design which does away with portions of the real errors in the experiment, be it noted, must be accompanied by their elimination in the statistical treatment of results. Unless this is realized, the increased precision becomes worse than useless, since, in being inevitably thrown away by an inappropriate test of significance, it impairs the validity of the conclusion" (Eugene Shen, "Experimental Design and Statistical Treatment in Educational Research," *Journal of Experimental Education*, VIII [1940], 348).

<sup>8</sup> These factors are: MA, IQ, initial rate, and initial accuracy of computation. The intracenter sections were also matched for CA in setting up the experiment; but measures of CA were omitted in the statistical treatment of the results as reported in chaps. iii-v.

<sup>9</sup> Shen's formula VI, *op. cit.*, p. 349.

### CHAPTER III

## THE EXPERIMENT IN CENTER A

The arithmetic program in Grades 1 and 2 is more extensive in Center A than that in Center C and less extensive than that in Center B. Besides this difference in relative amounts of arithmetic taught in the first two grades there is a difference as well in the relative emphasis put upon meaningful experiences as contrasted with abstract practice. In this respect, Center A schools surpassed the schools in the other two centers, especially those in Center C.

Center A, therefore, afforded an opportunity to find out the extent to which, if at all, a background of meaningful arithmetic conditions the relative effectiveness of DR, DM, EAR, and EAM. If the results in Center A closely approximate those in Center C, it should be safe to infer that the advantages and disadvantages of the subtraction procedures are little affected by the kind and amount of arithmetic learned earlier.

### SCHOOLS AND TEACHERS

Twelve teachers in seven schools took part in the experiment in Center A. All the teachers were experienced and capable, and all had volunteered for the experiment. Furthermore, each teacher was privileged to select the one of the four subtraction procedures she preferred to teach, if she had any preference. All experimental subjects within any one building were taught the same basic method, D or EA, never both.

In six schools the experiment began on November 11 and ended on December 3. In the seventh the experiment began a week later, but lasted, as in the other schools, for three school weeks (fifteen days of instruction).

### THE SUBJECTS

The initial number of subjects was 396, this total being reduced eventually to 328 as shown in Table 1. The enrollment of experimental subjects who provided usable data ranged from 78 (EAR) to 85 (DR).

The degree to which it was possible to equalize the four experimental sections with respect to factors considered of importance is

TABLE 1  
CENTER A—DISTRIBUTION OF SUBJECTS

| Sections   | Initial<br>Number<br>of Subjects | Subjects Lost         |                        |           | Number<br>of Subjects<br>Finally<br>Used |
|------------|----------------------------------|-----------------------|------------------------|-----------|------------------------------------------|
|            |                                  | Incomplete<br>Records | Process of<br>Equating | Repeaters |                                          |
| DR.....    | 109                              | 19                    | 3                      | 2         | 85                                       |
| DM.....    | 93                               | 4                     | 2                      | 5         | 82                                       |
| EAR.....   | 89                               | 2                     | 0                      | 9         | 78                                       |
| EAM.....   | 105                              | 8                     | 5                      | 9         | 83                                       |
| Total..... | 396                              | 33                    | 10                     | 25        | 328                                      |

shown in Tables 2 to 6. In the case of each factor an attempt was made to secure comparable means and standard deviations. This purpose was fairly well achieved. All differences in means were tested for reliability by the use of the "short" formula:<sup>1</sup>

$$C. R. = \frac{M_1 - M_2}{\sqrt{\sigma M_1^2 + \sigma M_2^2}}$$

Such differences are customarily regarded as reliable when the division in the formula produces a quotient of 3.00 or more.

#### *Comparability as to IQ*

Measures of IQ were assumed to be of primary importance in view of the fact that two of the subtraction procedures (DR and EAR) were to be taught in such a way as to make demands upon understanding. The Kuhlmann-Anderson Intelligence Test was administered to all children by experienced testers, and the test papers were scored twice to insure accuracy. The relevant data appear in Table 2.

In general, the differences among the sections in IQ are slight, the means for three of the sections varying within the range of 3.30 points. The mean for the EAM-section (99.39) is reliably higher than that for both the DM-and the EAR-sections. Moreover, this group of subjects was more homogeneous with respect to IQ than was any of the other three sections. It is therefore clear that what-

<sup>1</sup> As has already been explained (p. 41), differences in means in the case of the experimental results were tested by a formula which allows for correlation between factors. For the matching of the groups initially, however, it was felt that the simpler formula as above is entirely adequate.

TABLE 2  
CENTER A—COMPARABILITY OF SECTIONS WITH RESPECT TO IQ

| <i>Section</i>  | <i>M</i> | <i>σ<sub>dist.</sub></i> | <i>σ<sub>M</sub></i> |
|-----------------|----------|--------------------------|----------------------|
| DR (N=85).....  | 96.75    | 13.63                    | 1.50                 |
| DM (N=82).....  | 93.81    | 10.95                    | 1.21                 |
| EAR (N=78)..... | 93.45    | 11.30                    | 1.28                 |
| EAM (N=83)..... | 99.39    | 10.01                    | 1.10                 |

## CRITICAL RATIOS:

|          |      |           |      |
|----------|------|-----------|------|
| DR - DM  | 1.52 | DM - EAR  | 0.21 |
| DR - EAR | 1.08 | EAM - DM  | 3.42 |
| EAM - DR | 1.42 | EAM - EAR | 3.51 |

ever experimental advantages accrue to superior brightness favored the EAM-subjects.

*Comparability as to MA*

As shown in Table 3, the four experimental sections were well matched with respect to MA. The means vary within a range of about two months; the standard deviations of the distributions are much alike; and the differences in means are in no case reliable (see entries below the table). The advantage in IQ enjoyed by Section EAM is therefore considerably reduced by this tabulation, which employs measures of mental status instead of measures of brightness.

TABLE 3  
CENTER A—COMPARABILITY OF SECTIONS WITH RESPECT TO MA

| <i>Section</i>  | <i>M</i> | <i>σ<sub>dist.</sub></i> | <i>σ<sub>M</sub></i> |
|-----------------|----------|--------------------------|----------------------|
| DR (N=85).....  | 103.42   | 9.80                     | 1.08                 |
| DM (N=82).....  | 101.80   | 7.35                     | 0.81                 |
| EAR (N=78)..... | 102.64   | 8.05                     | 0.91                 |
| EAM (N=83)..... | 103.72   | 7.86                     | 0.86                 |

## CRITICAL RATIOS:

|          |      |           |      |
|----------|------|-----------|------|
| DR - DM  | 1.20 | EAR - DM  | 0.69 |
| DR - EAR | 0.55 | EAM - DM  | 1.63 |
| EAM - DR | 0.22 | EAM - EAR | 0.86 |

*Comparability as to CA*

The average chronological ages in the DR-, the DM-, and the EAR-sections were closely comparable, amounting to about 109 months in each case (Table 4). The children in the EAM-section were, however, considerably younger, as may have been surmised from the relationships already reported in the two preceding tables.

Their mean age was nearly five months below that in the other experimental sections, the differences in means being reliable in the case of DM- and EAR-pupils. Moreover, the variability in chronological age in the EAM-group was considerably less than in the other sections.

TABLE 4  
CENTER A—COMPARABILITY OF SECTIONS WITH RESPECT TO CA

| Section         | <i>M</i> | $\sigma_{dist.}$ | $\sigma_M$ |
|-----------------|----------|------------------|------------|
| DR (N=85).....  | 108.16   | 9.97             | 1.09       |
| DM (N=82).....  | 109.24   | 10.46            | 1.16       |
| EAR (N=78)..... | 109.99   | 9.66             | 1.09       |
| EAM (N=83)..... | 104.59   | 6.99             | 0.77       |

CRITICAL RATIOS:

|          |      |           |      |
|----------|------|-----------|------|
| DM - DR  | 0.68 | EAR - DM  | 0.47 |
| EAR - DR | 1.19 | DM - EAM  | 3.35 |
| DR - EAM | 2.68 | EAR - EAM | 4.06 |

So far, then, as IQ, MA, and CA are concerned—what may be called “general factors” in the experiment—the DR-, DM-, and EAR-subjects were closely similar; but the EAM-subjects were on the whole somewhat younger and brighter, though of the same mental status (Table 3).

*Comparability as to Computational Accuracy*

The experimental sections were matched, not alone with respect to what have been called “general factors” above, but also with respect to their skill in the kinds of abstract computation which had been taught prior to the start of the experiment. For this purpose the Pretest was planned and used. It consisted of a total of forty-nine examples, eighteen in addition (with and without carrying) and thirty-one in subtraction without borrowing. Two measures were secured: accuracy (number of correct answers) and rate of work (number of examples attempted within the time limits). Reliability of the test is indicated by the coefficient,  $r_{11} = .82$ .

The experimental sections were matched for accuracy in terms of their scores on the thirty-one subtraction examples only. The eighteen addition examples were interspersed with subtraction examples only to make the test typical of those to which the children were accustomed and to prevent their establishing a mental set to subtract which could operate throughout the test.

In spite of the liberal time allowance, the test seemed to have been sufficiently rigorous to yield good discrimination in accuracy of subtraction. The average score for all sections is about 20, and this is equivalent to about 65 per cent of the possibility. In only one section did as many as 15 per cent of the subjects secure scores of 30 or 31.

The facts reported in Table 5 show that the four experimental sections were well matched in subtraction without borrowing. The largest Critical Ratio (1.64 for EAM-DR) is far below the criterion (3.00), and only one other Critical Ratio (DM-DR) exceeds 1.00.

TABLE 5  
CENTER A—COMPARABILITY OF SECTIONS WITH RESPECT TO  
COMPUTATIONAL ACCURACY

| <i>Section</i>  | <i>M</i> | $\sigma_{dist.}$ | $\sigma M$ |
|-----------------|----------|------------------|------------|
| DR (N=85).....  | 19.46    | 7.40             | 0.81       |
| DM (N=82).....  | 20.76    | 6.72             | 0.74       |
| EAR (N=78)..... | 20.38    | 6.73             | 0.77       |
| EAM (N=83)..... | 21.18    | 6.69             | 0.67       |

CRITICAL RATIOS:

DM — DR 1.18  
EAR — DR 0.82  
EAM — DR 1.64

DM — EAR 0.36  
EAM — DM 0.42  
EAM — EAR 0.49

*Comparability as to Rate of Computation*

Rate scores were obtained by having the children stop and mark the example on which they were working at the end of eight minutes and at one-minute intervals thereafter until all except the few slowest had finished, or until fifteen minutes of testing time had elapsed.

TABLE 6  
CENTER A—COMPARABILITY OF SECTIONS WITH RESPECT TO  
RATE OF COMPUTATION

| <i>Section</i>  | <i>M*</i> | $\sigma_{dist.}$ | $\sigma M$ |
|-----------------|-----------|------------------|------------|
| DR (N=85).....  | 31.78     | 9.17             | 1.01       |
| DM (N=82).....  | 33.58     | 11.36            | 1.25       |
| EAR (N=78)..... | 34.34     | 8.62             | 0.98       |
| EAM (N=83)..... | 32.24     | 7.94             | 0.87       |

\*M equals number of examples attempted in 15 minutes.

CRITICAL RATIOS:

DM — DR 1.12  
EAR — DR 1.82  
EAM — DR 0.36

EAR — DM 0.48  
DM — EAM 0.88  
EAR — EAM 1.60

Admittedly, rate scores are not so valid measures of mastery as are accuracy scores: the fastest worker might score 0 in accuracy. Yet rate of work is an important aspect of performance. On this basis the experimental sections were fairly well matched, there being no Critical Ratio as large even as 2.00 (Table 6). It is to be noted that the fastest section, EAR, ranked third among the sections in accuracy (Table 5).

### *Summary on Initial Comparability*

As here measured, no difference in average achievement in subtraction computation, in either accuracy or rate, is reliable. In other words, on these critical bases the four experimental sections were about equal. On the more general bases, namely, CA, MA, and IQ, the DM-, DR-, and EAR-sections were also about equal, the EAM-children being somewhat younger and brighter.

### SYNOPSIS OF EXPERIMENTAL PROCEDURE

The classes in the four experimental sections were taught to borrow for a period of fifteen school days, each section according to its own instructional program as previously outlined (pp. 28-31). (1) Midway of the three-week period class papers were collected from the R-subjects only, to determine the extent to which the crutches taught were being used. (The M-sections were taught no crutches.) (2a) At the end of the fifteen days a computational test was administered. Part I of this test was restricted in content to the kind of subtraction just taught. Part II was similar in content to Part I, but the R-subjects were denied use of the crutch. Part III extended borrowing into new applications, with hundreds as well as with both tens and hundreds. It was designed to provide measures of transfer. Part I supplied data on the use of the crutches in R-classes. (2b) Immediately after the End Test had been given, all subjects were interviewed, to ascertain relative "smoothness" in computation and degree of understanding of the process. (3) About three weeks later R-subjects took a subtraction test, again to discover the degree to which the crutches were being used. (4a) Six weeks after the close of the experimental period proper a retention test was administered, intended to show progress or retrogression in the kind of subtraction taught and the extent of use of crutches by R-children. (4b) Immediately thereafter the second interviews were held with all subjects available, for the purposes mentioned in (2b) above.

## PLAN OF REPORT OF EXPERIMENTAL FINDINGS

In order to make the experimental findings intelligible, the chronological order of evaluation as described in the foregoing paragraph will be abandoned in favor of an organization around the following problems:

1. Process Taught: Accuracy of Computation
2. Process Taught: Rate of Computation
3. Process Taught: Smoothness of Performance
4. Process Taught: Degree of Understanding
5. Transfer to Untaught Processes: Accuracy of Computation
6. Transfer to Untaught Processes: Rate of Computation
7. Usefulness and Retention of Crutches

In connection with Problems 1-4 the test and interview data obtained both at the close of the experimental teaching and six weeks later will be considered together and in sequence, in order to complete the discussion of each problem.

*Process Taught: Accuracy of Computation**End Test, Part I*

Part I of the computation test given at the end of the fifteen-day instructional period comprised eighteen examples, fourteen of which involved borrowing. Four nonborrowing examples were scattered throughout the test in order to require a decision on the part of the children as to whether they should or should not borrow in each example. Table 7 contains a summary of the data for the test as a whole.

In accuracy the four subtraction procedures rank, from best to poorest: DR, EAR, EAM, and DM. DR was reliably superior to DM (by a wide margin) and to EAM, but not so to EAR. In turn, EAR was reliably superior to DM, but not to EAM. The showing for DM was indeed poor. To judge from these first data one might say that attempts to teach borrowing meaningfully pay dividends, particularly in the case of D. The superiority of D over EA is clear cut only when D is taught meaningfully or rationally and EA is taught mechanically.

Table 7 reports the accuracy results on Part I of the End Test as a whole. Separate analyses were made for the borrowing and the nonborrowing examples, as summarized in Tables 8 and 9, respectively.

TABLE 7  
CENTER A—END TEST, PART I: RELATIVE ACCURACY, WHOLE TEST  
(18 EXAMPLES)

| Section         | <i>M</i> | $\sigma_{dist.}$ | $\sigma M$ |
|-----------------|----------|------------------|------------|
| DR (N=82).....  | 14.88    | 3.74             | 0.41       |
| DM (N=79).....  | 12.28    | 5.45             | 0.61       |
| EAR (N=77)..... | 14.23    | 4.73             | 0.54       |
| EAM (N=80)..... | 13.78    | 4.22             | 0.47       |

## CRITICAL RATIOS:\*

|          |      |           |      |
|----------|------|-----------|------|
| DR — DM  | 8.13 | EAR — DM  | 5.42 |
| DR — EAR | 2.24 | EAR — EAM | 1.45 |
| DR — EAM | 4.08 | EAM — DM  | 4.41 |

\*As previously explained (pp. 40-41), the Critical Ratios in this table and in corresponding tables to follow were calculated according to the method proposed by Shen. See Appendix A, pp. 161ff.

TABLE 8  
CENTER A—END TEST, PART I: RELATIVE ACCURACY, BORROWING  
EXAMPLES (14 EXAMPLES)

| Section         | <i>M</i> | $\sigma_{dist.}$ | $\sigma M$ |
|-----------------|----------|------------------|------------|
| DR (N=82).....  | 11.56    | 3.05             | 0.34       |
| DM (N=79).....  | 9.89     | 4.69             | 0.53       |
| EAR (N=77)..... | 10.71    | 4.40             | 0.50       |
| EAM (N=80)..... | 10.81    | 3.72             | 0.43       |

## CRITICAL RATIOS:

|          |      |           |      |
|----------|------|-----------|------|
| DR — DM  | 5.96 | EAR — DM  | 2.48 |
| DR — EAR | 3.27 | EAM — DM  | 2.97 |
| DR — EAM | 3.13 | EAM — EAR | 0.34 |

TABLE 9  
CENTER A—END TEST, PART I: ACCURACY ON NONBORROWING  
EXAMPLES (4 EXAMPLES)

| Section  | Per Cents of Subjects Making Scores |    |    |    |    |
|----------|-------------------------------------|----|----|----|----|
|          | 4                                   | 3  | 2  | 1  | 0  |
| DR.....  | 59                                  | 28 | 6  | 10 | 6  |
| DM.....  | 28                                  | 23 | 23 | 14 | 13 |
| EAR..... | 60                                  | 25 | 12 | 1  | 3  |
| EAM..... | 44                                  | 25 | 19 | 7  | 5  |

On the borrowing examples alone, the DR-subjects were reliably superior to the subjects in the three other sections. The EAR- and the EAM-classes subtracted with equal accuracy on these examples, and both groups surpassed (but not reliably) the performance of the DM-children in this respect.

Interesting differences among the experimental sections were found in the case of the nonborrowing examples. Their comparative accuracy scores are shown in Table 9.

About 60 per cent of the DR- and of the EAR-pupils made perfect scores on the nonborrowing examples, and only 16 per cent and 4 per cent, respectively, of the two groups were in serious difficulty with examples of this type. The picture for the DM-children is one of confusion, a fourth of them making scores of 0 or 1. The EAM-subjects occupy an intermediate position, being less successful than the DR- and EAR-classes but more successful than the DM-classes.

Here is further evidence of the value of teaching so that children understand. The DR- and the EAR-subjects moved into the new process (borrowing) without interference with their skill in the old process (subtraction without borrowing). Understanding their new procedures, they apparently were able to discriminate well between the two types of subtraction, and they seem to have employed with each (both new and old) the appropriate techniques. By contrast, the DM-children (and to a less extent the EAM-children), lacking rational procedures with the new process, lost control over the old process. They were unable to distinguish well between the two types, and as a consequence made an undue number of errors with the type of subtraction which they had previously mastered.

### *Retention Test*

The Retention Test, given six weeks after the End Test and with very little intervening practice,<sup>2</sup> contained forty-two examples, thirty-two of them requiring borrowing from tens. Table 10 summarizes the accuracy scores for the four experimental sections on the test as a whole.

According to the means and Critical Ratios in Table 10 there was little to choose among the DR-, EAR-, and EAM-classes with respect to accuracy of subtraction. All three of these experimental sections, however, surpassed the record made by the DM-section. The deficiencies of these last subjects at the conclusion of the Experimental Period seem to have persisted, while the slight initial advantage of the DR-children at the earlier time was erased in the ensuing six weeks.

<sup>2</sup> The textbook used calls for only twenty-one subtractions during the period in question, divided eleven and ten between borrowing and nonborrowing types. The teachers in Center A followed the textbook closely, so much so that the statement about practice above is made with confidence.

TABLE 10  
CENTER A—RETENTION TEST: RELATIVE ACCURACY, WHOLE  
TEST (42 EXAMPLES)

| <i>Section</i>  | <i>M</i> | $\sigma_{dist.}$ | $\sigma M$ |
|-----------------|----------|------------------|------------|
| DR (N=74).....  | 33.22    | 10.56            | 1.23       |
| DM (N=79).....  | 28.32    | 12.64            | 1.42       |
| EAR (N=70)..... | 35.06    | 9.18             | 1.10       |
| EAM (N=76)..... | 34.13    | 8.34             | 0.96       |

## CRITICAL RATIOS:

DR — DM 5.70

EAR — DR 2.42

EAR — DM 8.12

EAR — EAM 1.41

EAM — DR 1.30

EAM — DM 7.26

TABLE 11  
CENTER A—RETENTION TEST: RELATIVE ACCURACY, BORROWING  
EXAMPLES (32 EXAMPLES)

| <i>Section</i>  | <i>M</i> | $\sigma_{dist.}$ | $\sigma M$ |
|-----------------|----------|------------------|------------|
| DR (N=74).....  | 25.00    | 8.76             | 1.02       |
| DM (N=79).....  | 20.52    | 11.43            | 1.29       |
| EAR (N=70)..... | 26.34    | 7.71             | 0.92       |
| EAM (N=76)..... | 25.50    | 6.69             | 0.77       |

## CRITICAL RATIOS:

DR — DM 5.89

EAR — DR 2.09

EAR — DM 7.97

EAR — EAM 1.56

EAM — DR 0.85

EAM — DM 7.11

Table 11 is for the accuracy scores on the thirty-two borrowing examples of the Retention Test. On this type of subtraction alone (the nonborrowing examples being excluded), the relative showing of the experimental sections is the same as for the test as a whole.

The data in Tables 10 and 11, and the statements made about their relationships, may be made somewhat clearer by recourse to percentage comparisons. On the fourteen borrowing examples at the end of experimental instruction the DR-children had an average score of 86 per cent; on the thirty-two borrowing examples of the Retention Test, an average score of 79 per cent. For the DM-section the corresponding per cents were 71 and 64; for the EAR-section, 77 and 82; and for the EAM-sections, 77 and 80.

It is not difficult to understand why the DM-children should have shown their considerable loss of thirteen percentage points; the

loss is consistent with the hypothesis, well established in psychological research, that retention is a function of initial mastery. The children in the DM-sections had not advanced very far, relatively speaking, in the course of their instruction. Hence their skill was subject to deterioration. Yet this explanation, however satisfactory for the DM-subjects, does not hold in the case of the EAM-subjects. These latter children also learned mechanically (if somewhat closer to mastery), but their accuracy improved in the weeks following instruction.

No explanation is at hand for the loss in accuracy experienced by the DR-children. On the hypothesis again that mastery, including understanding, makes for retention, these subjects were expected to show a gain rather than a loss on the Retention Test. By contrast, the EAR-subjects did show a gain. It may well be that the three weeks' period of experimental teaching was not long enough for the EAR-children to gain control of their procedure and that this control was furthered in the six weeks which followed. This reasoning to account for the improvement of the EAR-classes is not altogether gratuitous. In Grades 1 and 2 these children had acquired a meaningful basis for the arithmetic they learned. It is not unlikely that, not understanding EA any too well as it was taught during the experimental period, they extended their understanding by their own efforts during the ensuing weeks. (Yet it must be admitted that this hypothesis is not supported by increases in understanding as revealed on Interviews I and II. According to Table 19 the EAR-mean for understanding on Interview I was 2.49 as compared with a mean of 2.52 on Interview II, as shown in Table 20.)

The discussion in the foregoing paragraphs is admittedly speculative: the anticipated results did not always occur; why? Perhaps it is as well, in the absence of clarifying data, to leave the facts as they are. The DM-children retrogressed in accuracy during the postexperimental period; the DR-children retrogressed somewhat also, but were the equal in accuracy of the EAR- and the EAM-subjects; the EAR- and the EAM-children, particularly the former, gained in accuracy over the six weeks.

The last comparison to be made among the experimental sections in the matter of accuracy on the Retention Test has to do with the ten nonborrowing examples. The essential data are assembled in Tables 12 and 13. According to Table 13, the per cents of children making the high scores of 8, 9, and 10 combined are, for the different

sections: DR, 74; DM, 57; EAR, 81; and EAM, 80. The DR-children were perhaps a little less skilled than the EAR- and the EAM-children in nonborrowing subtraction; but all three groups of children were much superior to the DM-group. Their relative standing at the end of the experimental teaching had been altered not at all. The same relationships are revealed in the Critical Ratios of Table 12.<sup>3</sup>

TABLE 12

CENTER A—RETENTION TEST: RELATIVE ACCURACY, NONBORROWING  
EXAMPLES (10 EXAMPLES)

| <i>Section</i>  | <i>M</i> | <i>σ<sub>dist.</sub></i> | <i>σ<sub>M</sub></i> |
|-----------------|----------|--------------------------|----------------------|
| DR (N=74).....  | 9.05     | 2.44                     | 0.28                 |
| DM (N=79).....  | 8.42     | 2.19                     | 0.25                 |
| EAR (N=70)..... | 9.31     | 2.08                     | 0.25                 |
| EAM (N=76)..... | 9.24     | 1.84                     | 0.21                 |

CRITICAL RATIOS:

DR — DM 3.71

EAR — DR 1.63

EAR — DM 5.56

EAR — EAM 0.47

EAM — DR 1.19

EAM — DM 5.47

TABLE 13

CENTER A—RETENTION TEST: PER CENTS OF ACCURACY ON NONBORROWING  
EXAMPLES (10 EXAMPLES)

| Section         | Per Cents of Subjects Making Scores |    |    |    |   |    |   |   |   |   |   |
|-----------------|-------------------------------------|----|----|----|---|----|---|---|---|---|---|
|                 | 10                                  | 9  | 8  | 7  | 6 | 5  | 4 | 3 | 2 | 1 | 0 |
| DR (N=74).....  | 47                                  | 19 | 8  | 11 | 4 | 0  | 4 | 3 | 3 | 1 | 0 |
| DM (N=79).....  | 25                                  | 24 | 8  | 22 | 9 | 10 | 0 | 1 | 0 | 1 | 0 |
| EAR (N=70)..... | 45                                  | 25 | 11 | 7  | 1 | 5  | 4 | 1 | 0 | 0 | 0 |
| EAM (N=75)..... | 38                                  | 26 | 16 | 9  | 7 | 1  | 1 | 0 | 0 | 0 | 1 |

<sup>3</sup> The mean accuracy for DR on the Retention Test as a whole was 33.22 (Table 10). Their mean for the borrowing examples only was 25.00 (Table 11), and for the nonborrowing examples only was 9.05 (Table 12). The sum of the two submeans is therefore 34.05, which is 0.83 larger than the mean for the whole test. Similar discrepancies between the sums of the submeans and the means themselves were found for the other experimental sections: 0.62 for DM, 0.59 for EAR, and 0.61 for EAM. The tabulation of all data revealed the discrepancies to be statistical artifacts, resulting from the use of a class interval of 2 throughout. (The means and sums of submeans agreed when a class interval of 1 was employed). It is to be noted that the discrepancies in no way affect the validity of the comparisons made. The latter invariably are made among the experimental sections, one function at a time, and not *across* functions.

## Process Taught: Rate of Computation

## End Test, Part I

In taking Part I of the End Test, the children were instructed to encircle the examples on which they were working at the end of four minutes. Measures of rate of work were then secured by counting the number of completed examples, extrapolating when necessary to give measures of rate of work uniformly for a period of five minutes. The scores so obtained are summarized in Table 14.

TABLE 14  
CENTER A—END TEST, PART I: RELATIVE RATE OF WORK

| Section         | $M^*$ | $\sigma_{dist.}$ | $\sigma M$ |
|-----------------|-------|------------------|------------|
| DR (N=83).....  | 14.65 | 3.64             | 0.40       |
| DM (N=79).....  | 15.23 | 4.33             | 0.49       |
| EAR (N=77)..... | 16.49 | 4.25             | 0.48       |
| EAM (N=80)..... | 15.66 | 4.18             | 0.47       |

\*M=average number of examples completed in five minutes.

## CRITICAL RATIOS:

DM - DR 2.00

EAR - DR 6.57

EAR - DM 4.34

EAR - EAM 2.68

EAM - DR 3.48

EAM - DM 1.34

The results of the analysis show EAR to be the fastest of the four procedures, followed in order by EAM, DM, and DR. EAR is reliably superior in rate of work to both D-procedures and nearly so to EAM; and EAM is reliably superior to DR. DM is faster than DR, but not reliably so.

The findings just reported are confirmed by another kind of tabulation, in which the per cents of pupils completing the test in the various sections are compared: EAR, 51.9; EAM, 46.9; DM, 39.0; DR, 28.1.

The EA-procedure, whether taught meaningfully or mechanically, is thus seen to have been superior in center A to the D-procedure so far as rate of work is concerned. This conclusion is in complete accord with the findings of other investigators. Whether this advantage is to count heavily in favor of EA as the procedure to teach is, however, another matter.

One other aspect of the figures in Table 14 is deserving of comment. In the case of the EA procedure, use of the written crutch in the R-classes did not slow up work as compared with the M-classes,

which subtracted without the use of any crutch. In the case of the D-procedure, the written crutch in the R-classes seems to have retarded rate of work somewhat as compared with that in the M-classes; but, as already stated, the difference is unreliable.

### Retention Test

On the Retention Test the EAM-children completed an average of 28.87 subtraction examples in eight minutes (Table 15). In rate of work they greatly (reliably) exceeded the DM- and the EAR-subjects. They surpassed the DR-subjects as well, but by a difference that is unreliable. In turn, the DR-children were reliably faster than the EAR-children and were generally faster than the DM-children also. The DM- and the EAR-subjects worked at about the same rate.

TABLE 15  
CENTER A—RETENTION TEST: RELATIVE RATE OF WORK

| Section         | $M^*$ | $\sigma_{dist.}$ | $\sigma_M$ |
|-----------------|-------|------------------|------------|
| DR (N=74).....  | 27.65 | 11.51            | 1.34       |
| DM (N=79).....  | 25.73 | 8.61             | 0.97       |
| EAR (N=70)..... | 25.34 | 7.99             | 0.96       |
| EAM (N=75)..... | 28.87 | 8.07             | 0.93       |

\* $M$  = average number of examples completed in eight minutes.

#### CRITICAL RATIOS:

|          |      |           |      |
|----------|------|-----------|------|
| DR - DM  | 2.49 | EAM - DR  | 1.63 |
| DR - EAR | 3.00 | EAM - DM  | 4.98 |
| DM - EAR | 0.62 | EAM - EAR | 5.79 |

Changes in rate of work over the six-week period were as follows: on Part I of the End Test the DR-children subtracted 2.9 examples (both borrowing and nonborrowing) per minute; and they subtracted 3.5 examples (both types) on the Retention Test. The corresponding figures for DM are 3.0 and 3.2; for EAR are 3.3 and 3.2; and for EAM are 3.1 and 3.6. The largest gains in rate were made by DR- and EAM-subjects. The gain for the DM-subjects is slight, while the EAR-subjects made little if any change. Moreover, the initial advantage in rapidity of work which the EAR-classes enjoyed at the end of the instructional period had been completely wiped out by the comparatively larger gains of the other three groups of subjects.

*Process Taught: Smoothness of Performance*

Ratings for smoothness of performance were obtained from the interviews. These ratings had nothing to do with the question of a pupil's level of understanding, but related rather to the rhythm of his reactions. Moreover, each child was rated independently, so as to avoid confusion arising from differences in rate of reaction from child to child. Thus the child who worked slowly but smoothly secured the same rating as did the child who worked rapidly and smoothly.

A scale of four points was employed. The rating of "1" meant that the subject seemed to have habituated his process thoroughly; he subtracted evenly and promptly (according to his own rate) and without interruption. The rating of "4" was given the subject whose procedure was erratic and uncertain, evidencing confusion with both the ones' and tens' figures. Such a child hesitated irregularly, at times seeming to recall "rules" or "steps" he had mastered imperfectly. Intermediate degrees of smoothness were given values of "2" and "3." Performance with respect to smoothness was evaluated separately for each interview example.

*Interview I, End of Experimental Period*

Data on smoothness of performance were collected for five examples in all, three borrowing, two nonborrowing, at the time of the first interview following the period of experimental instruction. The data for the borrowing examples are assembled in Table 16. The first line of this table is read: The 81 DR-children subtracted in 68 instances with the highest quality of smoothness (rating 1); in 151 instances with quality 2, and in 24 instances with quality 3. Their mean level of smoothness of performance on these three examples was 1.82, obtained by (a) weighting the frequencies by their scale values (thus, 68, 302, 72), (b) adding these amounts, and (c) dividing the sum by 243 (81 subjects, 3 examples each).

The two M-procedures produced the greatest smoothness in performance (low means = smooth performance); and of these two, EAM was far and away the better according to this criterion. The poorest showing, and by a wide margin in every instance, was made by EAR (mean, 2.21, with the closest mean, for DR, 1.82).

As a whole, these measures and their relationships are not surprising. The M-children had been taught language patterns which were reduced to a minimum of verbiage. On the other hand, the

TABLE 16

CENTER A—INTERVIEW I: SMOOTHNESS OF PERFORMANCE, BORROWING  
EXAMPLES (3 EXAMPLES)

| Section         | Frequency of Scores* |     |    |   | <i>M</i> | $\sigma_{dist.}$ | $\sigma M$ |
|-----------------|----------------------|-----|----|---|----------|------------------|------------|
|                 | 1                    | 2   | 3  | 4 |          |                  |            |
| DR (N=81).....  | 68                   | 151 | 24 | 0 | 1.82     | 0.59             | 0.04       |
| DM (N=79).....  | 114                  | 88  | 31 | 4 | 1.68     | 0.76             | 0.05       |
| EAR (N=77)..... | 18                   | 153 | 53 | 7 | 2.21     | 0.62             | 0.04       |
| EAM (N=79)..... | 172                  | 46  | 11 | 8 | 1.39     | 0.73             | 0.05       |

\*Low scores signify superior performance.

## CRITICAL RATIOS:

DR — EAR 7.80

DM — DR 2.46

DM — EAR 9.14

EAM — DR 7.54

EAM — DM 4.53

EAM — EAR 14.14

language patterns for the R-children were much more involved, for they carried the full story of the process. It is clear that the R-children, whether taught D or EA, were unable in the period of three weeks to arrive at smooth, rhythmical forms of thinking and expression in working with borrowing examples.

It is a safe assumption that at the start of the experiment all four sections were approximately equal in smoothness of performance when they dealt with nonborrowing examples. Conceivably the mean scale value of their performance might have been 1.0. With allowance for individual variations, the mean for all sections may be set as 1.1 or 1.2. Now, what happened to their relative smoothness of performance on nonborrowing examples *after* they had had experience with borrowing examples as well? Table 17 summarizes the facts.

TABLE 17

CENTER A—INTERVIEW I: SMOOTHNESS OF PERFORMANCE, NONBORROWING  
EXAMPLES (2 EXAMPLES)

| Section  | Frequency of Scores |    |   |   | Mean | $M_{Borrow-}$<br>ing —<br>(Table 16) | $M_{Non-}$<br>borrow-<br>ing |
|----------|---------------------|----|---|---|------|--------------------------------------|------------------------------|
|          | 1                   | 2  | 3 | 4 |      |                                      |                              |
| DR.....  | 126                 | 29 | 2 | 5 | 1.30 | 0.52                                 |                              |
| DM.....  | 103                 | 46 | 7 | 2 | 1.42 | 0.27                                 |                              |
| EAR..... | 93                  | 51 | 5 | 5 | 1.50 | 0.71                                 |                              |
| EAM..... | 133                 | 17 | 5 | 3 | 1.21 | 0.18                                 |                              |

On the assumptions proposed in the preceding paragraph, the EAM-children maintained their same level of performance, or slowed up inconsiderably, their mean on the two nonborrowing examples being 1.2. The subjects in all other sections, however, lost in smoothness of performance, especially those in DM- and EAR-classes. In other words, for them mastery of the new process of borrowing had retroactive influence on the smoothness of their performance in dealing with previously learned nonborrowing subtraction.

In the course of the interview the subjects encountered both borrowing and nonborrowing examples, and the two types were intermingled. On the nonborrowing examples the sections ranked in smoothness of performance, best to poorest, EAM, DR, DM, and EAR. EAM-subjects still made the best showing, and EAR-subjects the poorest.

The last column in Table 17 shows the "adjustment" in smoothness of performance in passing from nonborrowing to borrowing subtraction. The EAM-children were little disturbed: they lost but 0.18 points in scale value (1.39-1.21). By contrast, the EAR-children were much affected by change in type of subtraction. As already stated, they were the most irregular in performance in nonborrowing subtraction; yet they exceeded all other sections in the magnitude of their "adjustment" to borrowing examples (0.71). The DR-subjects showed the next largest magnitude of difference, followed by the DM-subjects.

To summarize the findings from Interview I, the EAM-classes worked more smoothly than did the other experimental sections at the time of Interview I. Moreover, their average in smoothness was about the same as it had been before undertaking borrowing subtraction, and they were the least affected in the interview by changes from nonborrowing to borrowing subtraction, and back again. At the opposite extreme were the EAR-children. Introduction of borrowing seems to have interfered greatly with the smoothness of their performance both on the new borrowing and on the old nonborrowing types of subtraction. The DR-and the DM-classes occupy intermediate positions in these comparisons.

### *Interview II*

The Interview II data on smoothness of performance are presented for both borrowing and nonborrowing examples in Table 18. In borrowing subtraction all three of the other sections surpassed

the EAR-section, reliably so in the case of EAM and DM, and nearly so in the case of DR. Their relative ranks were EAM, DM, DR, and EAR, the same as at the time of Interview I.

TABLE 18

CENTER A—INTERVIEW II: SMOOTHNESS OF PERFORMANCE,  
BORROWING AND NONBORROWING EXAMPLES

| Section       | Borrowing Examples  |    |    |    |      |                  |            | $M$ Non-<br>borrow-<br>ing Ex-<br>amples | $M$ Borrow-<br>ing —<br>$M$ Non-<br>borrow-<br>ing |
|---------------|---------------------|----|----|----|------|------------------|------------|------------------------------------------|----------------------------------------------------|
|               | Frequency of Scores |    |    |    | $M$  | $\sigma_{dist.}$ | $\sigma M$ |                                          |                                                    |
|               | 1                   | 2  | 3  | 4  |      |                  |            |                                          |                                                    |
| DR (N=79) ..  | 142                 | 74 | 18 | 3  | 1.50 | 0.69             | 0.05       | 1.29                                     | 0.21                                               |
| DM (N=77) ..  | 151                 | 60 | 20 | 0  | 1.43 | 0.65             | 0.04       | 1.32                                     | 0.11                                               |
| EAR (N=72) .. | 103                 | 92 | 10 | 11 | 1.67 | 0.79             | 0.05       | 1.30                                     | 0.37                                               |
| EAM (N=81) .. | 180                 | 51 | 6  | 6  | 1.33 | 0.65             | 0.04       | 1.28                                     | 0.05                                               |

## CRITICAL RATIOS:

DR — EAR 2.43

DM — DR 1.17

DM — EAR 3.43

EAM — DR 2.83

EAM — DM 1.67

EAM — EAR 4.86

All four sections attacked nonborrowing subtraction with comparable regularity (the means vary within a range of 0.04 scale points). The shift from nonborrowing to borrowing subtraction occasioned little or no need for "adjustment" for the EAM-subjects (0.05, last column), a slight "adjustment" for DM-subjects, a larger one for DR-subjects, and a major one for EAR-subjects.

The conclusion seems to be warranted that six weeks after the cessation of instruction the EAM-procedure had been well mastered, so far as rhythmic performance is concerned; the DM-procedure and the DR-procedure were less well mastered; and the EAR-procedure was still considerably short of mastery in borrowing subtraction.

*Process Taught: Degree of Understanding*

Measures of degree of understanding of the process of borrowing were obtained through interviews at the end of the experimental teaching and six weeks later. The technique of the interview has already been briefly described in pages 38-39 and is set forth in greater detail in Appendix B, pages 181-186. It is sufficient here to say that degree of understanding was evaluated on a five-point scale, ranging from "1" (complete understanding, with ability to explain all "steps" in borrowing subtraction) to "5" (utter confusion, with the use of "explanations" which accounted for nothing).

## Interview I

The measures of understanding reported for Interview I in Table 19, as well as the relationships between these measures, could perhaps have been anticipated. The procedures here denominated DR and EAR produced degrees of understanding consistent with the purpose for which they were designed. On the other hand, the DM- and the EAM-procedures, which were taught mechanically, left pupils with but a very slight grasp on the rationale of borrowing—this in spite of the fact that these particular subjects had, up to the start of the experiment, been taught arithmetic in a relatively meaningful way. Clearly, at the time these measures were secured, this meaningful background had functioned little in affording insight into borrowing on the part of the children taught EAM and DM.

TABLE 19  
CENTER A—INTERVIEW I: DEGREE OF UNDERSTANDING, BORROWING  
EXAMPLES (3 EXAMPLES)

| Section         | Frequency of Scores* |    |    |     |    | <i>M</i> | $\sigma_{dist.}$ | $\sigma_M$ |
|-----------------|----------------------|----|----|-----|----|----------|------------------|------------|
|                 | 1                    | 2  | 3  | 4   | 5  |          |                  |            |
| DR (N=81).....  | 130                  | 19 | 33 | 51  | 10 | 2.14     | 1.36             | 0.09       |
| DM (N=79).....  | 1                    | 0  | 6  | 186 | 44 | 4.15     | 0.48             | 0.03       |
| EAR (N=77)..... | 68                   | 57 | 50 | 37  | 19 | 2.49     | 1.29             | 0.09       |
| EAM (N=79)..... | 26                   | 15 | 32 | 144 | 20 | 3.49     | 1.10             | 0.07       |

\*Low scores signify superior performance.

## CRITICAL RATIOS:

DR — DM 23.10  
DR — EAR 2.92  
DR — EAM 12.27

EAR — DM 18.44  
EAR — EAM 9.09  
EAM — DM 8.25

In supplying understanding of the process DR excelled the other three procedures. The difference between the DR-mean and the DM- and the EAM-means is overwhelmingly large, and the difference with the EAR-mean is, for all intents, reliable as well. The record for EAR as compared with DM and EAM is likewise impressive. Of the four procedures that of DM was especially bad so far as understanding is concerned, its mean being reliably inferior even to that for EAM, which was also taught mechanically.

In so far, then, as understanding is an important factor in initial learning, borrowing, whether taught by D or by EA, should be taught with emphasis upon relationships, and DR is to be preferred to EAR. If D is taught, then certainly it should be taught as DR, for the

showing of DM is poor indeed. If understanding in the initial stages of learning is unimportant, then borrowing should be taught as EA rather than as D. This last statement is of considerable significance in the light of previous research findings, in which EA has been regularly reported as superior to D. In this earlier research both procedures were apparently taught mechanically, and the critical evaluative measures were those of rate and accuracy of work. The same superiority for EA over D, when both were taught mechanically, is here found in data relating to understanding.

The relative records made by DR- and by EAR-subjects are significant at another point. The claim has been made that EA is as easily taught as is D (see p. 8, above). To the extent that measures of understanding also measure ease of learning, the claim seems to be spurious: EA taught meaningfully for three weeks, even to pupils who had relatively sound backgrounds in meaningful arithmetic, was not as completely understood as was D taught under the same conditions.

### Interview II

The comparative standings of the four sections with respect to understanding at the time of Interview II are portrayed in Table 20. The rank of the sections is the same as at the time of Interview I: DR, EAR, EAM, and DM. All differences in means are reliably superior for DR; EAR is reliably superior to EAM and DM; and EAM, to DM. All except the EAM-subjects showed either improvement or maintenance of status in understanding. The change for DR-classes is .23 scale points (2.14-1.91); for EAR, .03, and for DM, .14. The EAM-children retrogressed .20 scale points.

TABLE 20  
CENTER A—INTERVIEW II: DEGREE OF UNDERSTANDING, BORROWING  
EXAMPLES (3 EXAMPLES)

| Section         | Frequency of Scores |    |    |     |    | <i>M</i> | $\sigma_{dist.}$ | $\sigma M$ |
|-----------------|---------------------|----|----|-----|----|----------|------------------|------------|
|                 | 1                   | 2  | 3  | 4   | 5  |          |                  |            |
| DR (N=79).....  | 130                 | 40 | 34 | 24  | 9  | 1.91     | 1.20             | 0.08       |
| DM (N=77).....  | 3                   | 4  | 13 | 179 | 32 | 4.01     | 0.62             | 0.04       |
| EAR (N=72)..... | 62                  | 52 | 48 | 35  | 19 | 2.52     | 1.29             | 0.06       |
| EAM (N=81)..... | 12                  | 13 | 36 | 160 | 22 | 3.69     | 1.34             | 0.09       |

CRITICAL RATIOS:

DR - DM 26.25  
DR - EAR 5.08  
DR - EAM 17.80

EAR - DM 14.90  
EAR - EAM 10.26  
EAM - DM 4.00

The lapse of time had not enabled the children taught mechanically—or at least it had not caused them—to discover the rationale of their borrowing procedures. Nor had the lapse of time lessened the advantage of DR over EAR in ease of understanding. On the contrary, it had markedly increased its degree of advantage. Here is further evidence, therefore, that EA as a rational process is more difficult to teach meaningfully than is D, even though both processes are taught to children who come to them with relatively strong arithmetical backgrounds.

*Transfer to Untaught Processes: Accuracy of Computation*

Instructional procedures are not adequately evaluated when the only measures obtained have to do with proficiency in the process taught. Of equal importance are measures of transfer. Other things being equal, procedures taught in connection with skill (*a*) are preferable to other procedures for teaching that skill when they make it easier for children to learn more advanced skills, (*b*) and (*c*). The experimental subjects were taught to borrow from tens only (skill *a*). What were the relative degrees of success for the four sections when they encountered borrowing from hundreds only (skill *b*) and from both hundreds and tens (skill *c*)?

Part III of the End Test contained eighteen subtraction examples. These were arranged in three rows; one example per row involved no borrowing. Seven examples required borrowing from hundreds only; eight from both hundreds and tens.

The accuracy scores for the four sections are summarized in Table 21 for the test as a whole. This table reveals one striking fact, namely, the vast superiority of DR as making for transfer to untaught processes. The DR-mean is superior to the three other section means by amounts of extraordinary size, the Critical Ratio in each case being larger than 12.0. Clearly, the children taught this procedure acquired understandings which functioned well under changed conditions of subtraction. The transfer records for the DM- and the EAM-children might perhaps have been predicted from the fact that they used procedures which afforded no insight into the rationale of borrowing. But the record of the EAR-children might not have been predicted, at least by advocates of EA who insist that it is no harder to teach meaningfully than is the D-procedure. The explanation for the difficulty experienced by the EAR-subjects in new types of borrowing has already been advanced in

connection with Table 19. It was there shown, on the basis of data collected in Interview I, that the EAR-children did not in three weeks attain the degree of understanding of their procedure that was attained by the DR-children with theirs. Here, then, is additional evidence that children profit much more from a brief period of meaningful instruction on D than on EA.

TABLE 21  
CENTER A—TRANSFER TEST: ACCURACY, WHOLE TEST  
(18 EXAMPLES)

| <i>Section</i>  | <i>M</i> | <i>σ<sub>dist.</sub></i> | <i>σ<sub>M</sub></i> |
|-----------------|----------|--------------------------|----------------------|
| DR (N=83).....  | 9.98     | 5.18                     | 0.57                 |
| DM (N=79).....  | 5.13     | 4.30                     | 0.48                 |
| EAR (N=77)..... | 5.44     | 4.65                     | 0.53                 |
| EAM (N=80)..... | 5.05     | 3.77                     | 0.42                 |

## CRITICAL RATIOS:

DR — DM 13.47

DR — EAR 12.27

DR — EAM 14.94

DM — EAM 0.26

EAR — DM 0.89

EAR — EAM 1.22

The separate tabulations made for borrowing and nonborrowing subtraction as summarized in Tables 22 and 23 are not reported in full. On the borrowing examples the means were: DR, 7.49; DM, 3.11; EAR, 2.83; EAM, 2.65 (Table 22). As would be expected from the data in Table 21, the DR-subjects were much superior to those in the three other sections, all the differences being reliable. The EAM-subjects were the poorest. The DM-subjects surpassed slightly (unreliably) the EAR-subjects, as they had not done on the test as a whole.

TABLE 22  
CENTER A—TRANSFER TEST: ACCURACY, BORROWING EXAMPLES  
(15 EXAMPLES)

| <i>Section</i>  | <i>M</i> | <i>σ<sub>dist.</sub></i> | <i>σ<sub>M</sub></i> |
|-----------------|----------|--------------------------|----------------------|
| DR (N=83).....  | 7.49     | 4.69                     | 0.52                 |
| DM (N=79).....  | 3.11     | 3.57                     | 0.40                 |
| EAR (N=77)..... | 2.83     | 4.18                     | 0.48                 |
| EAM (N=80)..... | 2.65     | 3.21                     | 0.36                 |

## CRITICAL RATIOS:

DR — DM 12.17

DR — EAR 13.71

DR — EAM 15.61

DM — EAR 0.82

DM — EAM 1.48

EAR — EAM 0.62

TABLE 23

CENTER A—TRANSFER TEST: PER CENTS OF ACCURACY, NONBORROWING  
EXAMPLES (3 EXAMPLES)

| Section  | Per Cents of Subjects Making Scores |    |    |    |
|----------|-------------------------------------|----|----|----|
|          | 3                                   | 2  | 1  | 0  |
| DR.....  | 67                                  | 21 | 6  | 6  |
| DM.....  | 47                                  | 24 | 13 | 16 |
| EAR..... | 73                                  | 15 | 8  | 4  |
| EAM..... | 58                                  | 30 | 4  | 8  |

In nonborrowing subtraction (Table 23) the highest possible score was 3. This score was made by the following per cents of subjects in the different sections: DR, 67; DM, 47; EAR, 73; and EAM, 58. As was noted in the case of the End Test, familiarity with the crutch seems to have enabled the R-subjects to hold relatively steady their control over nonborrowing subtraction. The showing of the DM-children was, at the time of the End Test, especially unsatisfactory.

*Transfer to Untaught Processes: Rate of Comparison*

Table 24 permits a comparison of the four sections with respect to rate of work on the transfer test. The EAR-subjects attempted an average of 17.39 examples within the time limits. Their rate of work was much faster than that of the children in the three other sections, among which there was little difference in this respect.

TABLE 24

CENTER A—TRANSFER TEST: RELATIVE RATE OF WORK  
(18 EXAMPLES)

| Section         | <i>M</i> | $\sigma_{dist.}$ | $\sigma M$ |
|-----------------|----------|------------------|------------|
| DR (N=83).....  | 14.37    | 3.83             | 0.42       |
| DM (N=79).....  | 14.78    | 4.09             | 0.46       |
| EAR (N=77)..... | 17.39    | 4.51             | 0.51       |
| EAM (N=80)..... | 14.35    | 4.94             | 0.55       |

## CRITICAL RATIOS:\*

DR — EAM 0.06  
DM — DR 1.37  
DM — EAM 1.23

EAR — DR 9.44  
EAR — DM 7.68  
EAR — EAM 8.22

\*The coefficient of multiple correlation used in calculating Critical Ratios was the same as that for rate of work on Part I of the test (Table 14).

As a matter of fact, 70 per cent of the EAR-subjects finished before the time limits, as compared with 38 per cent for the EAM-section, 35 per cent for the DM-section, and 28 per cent for the DR-section. The writing of the crutch failed to slow up the EAR-subjects as compared with the EAM-subjects; quite the contrary. Nor did the writing of the crutch slow up the DR-subjects as compared with the DM-subjects. Just why the use of the EAR-crutch should have contributed so much more than the use of the DR-crutch to rate of work, it is impossible to say; but the findings here in connection with the Transfer Test agree with those in connection with the test of the process taught (Part I of the End Test).

### *Usefulness and Persistence of the Crutches*

The crutches taught the two R-sections are illustrated at the right. It was supposed that these devices were necessary to make the procedures fully meaningful. The procedures might have been taught through purely verbal terms, each with its own language pattern. These crutches, however, supplemented the language patterns with *perceptual* patterns, which showed concretely and visually the rationale of the procedures. At least such was the assumption.

Crutches of the kind taught in this investigation are usually opposed on the grounds that: (a) they are themselves difficult to learn, sometimes being more difficult than the processes they are expected to simplify; (b) once learned, they actually contribute little to efficiency; and (c) once learned, they become permanent parts of the total process; that is, they are seldom discarded. In so far as data are available from the present study, these issues will be considered under two heads: usefulness of the crutches [points (a) and (b) above] and persistence [(c) above].

#### *a. Usefulness of the Crutches*

The crutches were integral parts of the procedures taught. Hence it is impossible to isolate their influence from the procedures as wholes, in order to assess their peculiar values. The evidence on the usefulness of the crutches must therefore be negative in a sense. Whenever, on any measure, the DR-subjects surpassed the DM-subjects, and the EAR-subjects surpassed the EAM-subjects—whenver these conditions are met, it is inferable that the crutches did not

| <u>DR</u>    | <u>EAR</u>   |
|--------------|--------------|
| 8 7 12       | 8 7 12       |
| <u>4 5 9</u> | <u>4 5 9</u> |
| 4 1 3        | 4 1 3        |

interfere with learning and may actually have contributed to learning. Contrariwise, whenever the DM-subjects and the EAM-subjects surpassed their counterparts, it is inferable that the crutches were of little use and may have interfered with learning.

*The DR-Crutch.*<sup>3</sup> The DR-children greatly exceeded the DM-children in accuracy of computation, both on borrowing and on non-borrowing examples, and both at the end of the instructional period and six weeks later (Tables 7-13). The DM-children, after starting borrowing subtraction, seem to have lost control over nonborrowing subtraction to a far greater extent than did the DR-children.

The relationship between the sections in rate of subtraction was not quite the same. The DM-children excelled the DR-children in this respect on the End Test but were inferior to them on the Retention Test (Tables 14 and 15), in both instances by unreliable differences. It took time to write out the DR-crutch. The DR-children were required to show the crutch on their papers on the End Test, and hence were understandably slower. After the End Test the DR-children were directed to abandon the writing out of the crutch in their work. Under these conditions their relative rate of work was enhanced on the Retention Test. The conclusion seems to be warranted that, both in accuracy and in rate of work in borrowing from tens, the crutch imposed no special difficulties upon the DR-children and probably helped them materially in learning the new process.

In smoothness of performance the DM-children were (unreliably) superior both at the time of the End Test (C.R., 2.5, Table 16) and at the time of the Retention Test (C.R., 1.2, Table 18). In degree of understanding a very large advantage lay with the DR-children both on Interview I and on Interview II (C.R.'s, 23.1 and 26.3, Tables 19 and 20).

According to Tables 21 and 22, the DR-subjects outscored the DM-subjects in transferring their skills to untaught processes. This statement holds for accuracy of computation, but not for rate, in which the DM-children were slightly superior. The latter aspect of skill (rate) is, however, much less important than is accuracy, particularly when rate is accompanied by loss in accuracy.

<sup>3</sup> The DR-crutch was investigated, though less comprehensively, in an earlier study which yielded results very much like those reported in the present research. See William A. Brownell, *et al.*, *Learning as Re-Organization* (Duke University Research Studies in Education, No. 3. Durham, N. C.: The Duke University Press, 1939).

In summary, and on the assumptions previously stated, the crutch taught the DR-children appears to have helped them very greatly indeed. Only on relatively minor phases of subtraction did the DM-children manifest any superiority.

*The EAR-Crutch.* The case for the EAR-crutch is not so convincing. In accuracy of computation in the process taught, the EAR-children and the EAM-children were on about equal terms; in no instance is the difference in their means on the End Test and on the Retention Test a reliable one (Tables 7-12). If the crutch did not help, neither did it hinder.

In rate of computation, curiously enough the EAR-children were superior on the End Test when they had to write out their crutch, but inferior on the Retention Test when they did not have to.

In smoothness of performance the EAM-children excelled on both Interview I and Interview II (Tables 16 and 18). In degree of understanding revealed in the two interviews, however, the EAR-subjects earned very much larger scores (Tables 19 and 20).

On the Transfer Test the two sections were about equal in accuracy (Table 21), but the EAR-children subtracted very much faster (Table 24).

On the whole, then, there is some evidence that the EAR-crutch assisted its learners, and there is little or no evidence that it interfered with their learning.

One can only conjecture with regard to the failure of the EAR-crutch to yield less ambiguous evidence of its value than was produced by the DR-crutch. It is of course apparent from a comparison of the two crutches (see p. 65) that the EAR-crutch is a bit more complicated than is the DR-crutch. Two alterations must be made in the borrowing example, and the second of these (in the minuend) can be made only with some difficulty in examples which are set up for subtraction. But the explanation may not be associated with this condition. It may very well be true that the crutch used (the best one which could be devised) does not tie in as closely with the rationale of borrowing as does the DR-crutch. Some support for this hypothesis is found in the fact that the DR-children understood borrowing by their procedure better than did the EAR-children by their procedure on both interviews (C.R.'s 2.9 and 5.1, for Interviews I and II, respectively, Tables 19 and 20).

*Summary.* The objections commonly raised to the kinds of learning aid taught in this experiment do not hold for the two crutches

used. No positive claim can be made that the EAR-crutch was helpful; but the data indicate that, contrary to the criticisms of such devices, it did no harm. For the DR-crutch positive claims of helpfulness seem to be inferable from the data.

*b. Persistence*

Perhaps the simplest means by which to trace the history of crutch use is that of graphs (Figs. 1 and 2). The base lines in both graphs contain five points in time. The first three are for the period of instruction when the crutches were *required* in written work. That is to say, 100 per cent use was sought and was expected. The last two points were for the Postexperimental Period, during which steps were taken to discourage further use of the written crutches.

At the close of the period of experimental teaching, the teachers of crutch sections were given the following directions:

You are to deny your pupils use of the crutch from the start of the postexperimental period onward. The procedure is to illustrate a "short way" to subtract without the device and to demonstrate that the crutch is really no longer needed. For a few lessons whenever you assign seat-work in subtraction remind the children not to use the crutch. If you find any pupils continuing to rely upon it, find out why and try to teach them away from it.

During the Experimental Period proper two sets of papers were collected from the crutch sections. The first set was for the sixteen borrowing examples on page 87 of the text, which was assigned at the end of the first week. The second set was for page 98, which contained twenty-two borrowing examples and was assigned more than a week later. Part I of the End Test contained fourteen borrowing examples. A special test, given at the middle of the Postexperimental Period, contained fourteen borrowing examples and was scored for use of the crutches only. The Retention Test contained thirty-two borrowing examples.

The curves in Figure 1 are for per cents of use. Seventy-seven DR-children each had sixteen chances to use the crutch on the first practice page. There was therefore a total of 1232 chances to use the DR-crutch. The crutch was actually used 1211 times, for a per cent of 98. All other per cents in Figure 1 were obtained in the corresponding manner.

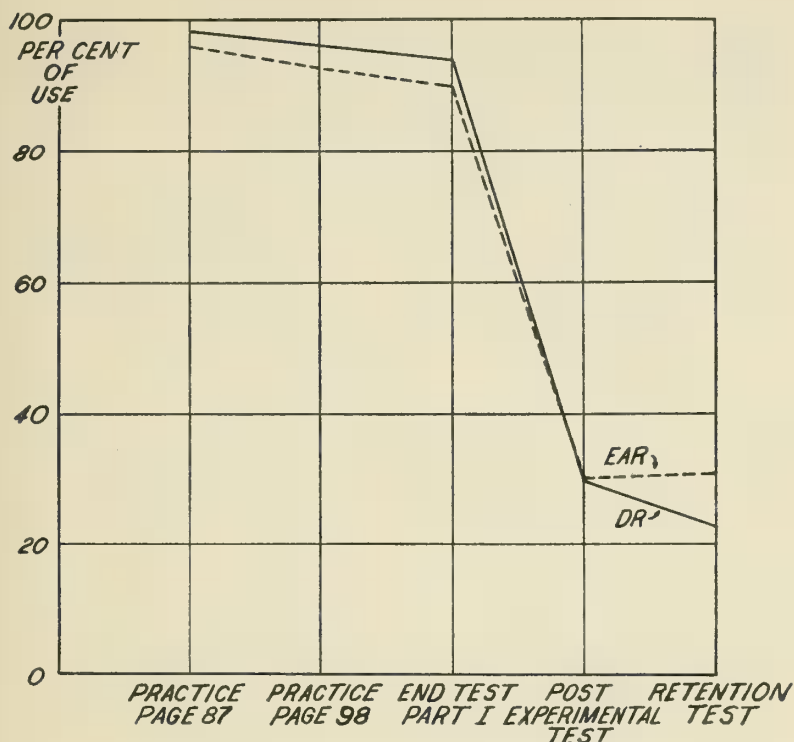


FIG. 1. Actual number of crutch uses divided by possible number of uses, at successive times of measurement.

The DR- and the EAR-curves are closely similar. Both show a high degree of use of crutches during the instructional period, somewhat higher in the case of DR; but they do not show 100 per cent use. In both instances there was a slow but steady abandonment of the crutch during the instructional period. The effect of new directions from the teachers, directions designed to eliminate the crutches, is shown in the rapid drop to approximately 30 per cent at the time of the test at the middle of the Postexperimental Period. Thereafter the DR-children tended to continue to discard their crutch, but not the EAR-children.

Figure 2 is differently constructed, to show the per cents of children who used the crutches. It is to be noted that if a child used a crutch even once on any practice page or test, he was counted as a user of the crutch.

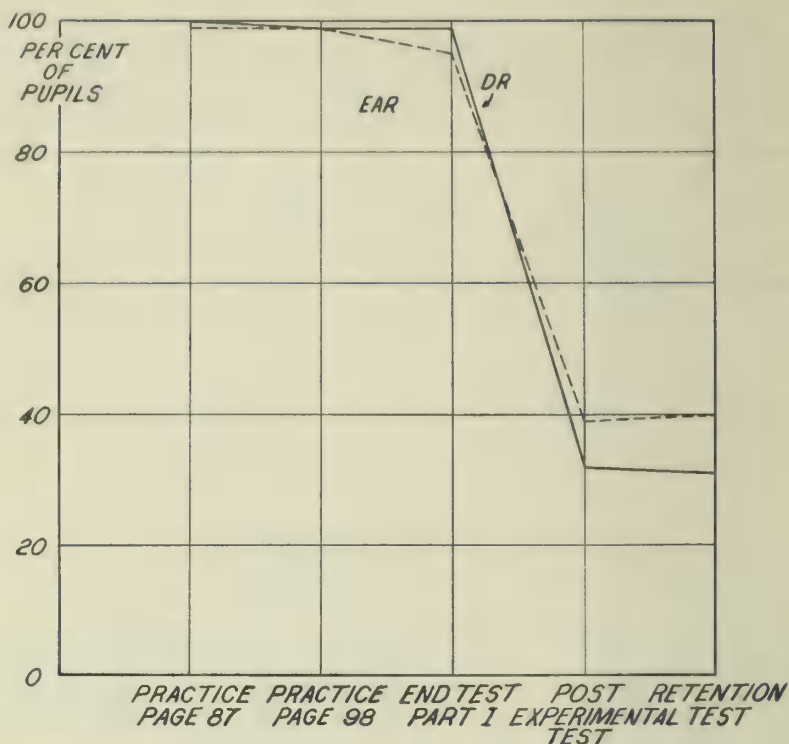


FIG. 2. Per cents of pupils using crutches, at successive times of measurement.

During the period of instruction the per cents of users never fell below 95. At the middle of the Postexperimental Period, and after directions to discard the crutches, the per cents fell abruptly to about 40 (EAR) and 35 (DR). In the three weeks following there was little or no change in the case of EAR, and a very slight drop in the case of DR.

Whichever procedure is employed to measure persistence, the data reveal a trend away from the crutches. This trend is somewhat more marked for DR than for EAR. An examination of the records for the EAR-classes shows that continued frequent use of the crutch was confined rather largely to a single class. It is fair to infer that the teacher in this class was not as insistent as were the other two teachers in this section upon abandonment of the device.

The subjects in this experiment were slower to reject their crutches than were the subjects in an earlier investigation<sup>4</sup> in which

<sup>4</sup> Brownell, *op. cit.*

the DR-crutch alone was taught. No explanation for the difference in findings can be advanced with assurance. It may be that the present corps of teachers did not work as hard for the abandonment of the crutches. It may be that the smaller amount of borrowing practice allowed in the Postexperimental Period in the present study militated against abandonment of the crutch. But whatever the explanation, a number of questions remain: What of it? Was lasting harm produced by continued use of the crutches? On the other hand, if the children needed the crutches, should they not have been allowed to use them? And, last of all, how much longer would the children who employed the crutches on the Retention Test have persisted in their use?

One troublesome problem in teaching crutches is to determine when these devices should be denied children in favor of shorter and more direct procedures. In the present instance this time was set arbitrarily three weeks after introduction of the crutches. This may have been too soon, or it may have been too long delayed. Data are available to supply at least a partial answer. It will be recalled that on Part II of the End Test the crutch children were directed to subtract without writing out the devices. Under these conditions the DR-children averaged about 1.5 points lower in accuracy than on Part I. That is to say, they secured 1.5 fewer correct answers in the same length of time. Moreover, they subtracted somewhat slower, to the extent of about 0.75 example within the time limits. The EAR-children's mean in accuracy for Part II suffered a loss of 0.7 points, but in rate of work an improvement of 0.6. These changes<sup>5</sup> are all within the range of error of measurement, and it may be inferred that the crutch children performed on Part II *without* their crutches as well as they did on Part I *with* them. Hence, use of the crutches does not seem to have been withdrawn prematurely.

#### SUMMARY AND INTERPRETATION

Before the conclusions from the experiment in Center A are summarized, two cautions should be recognized. One caution arises from the nature of the subjects of the experiment. It must be kept in mind that these particular children had had a rather full curriculum in arithmetic in Grades 1 and 2 and that some systematic effort

<sup>5</sup> The DM-pupils and the EAM had the same instructions for Part II as for Part I. For the DM-subjects the mean for accuracy dropped 0.65 points and for rate dropped 0.3 points. The corresponding figures for the EAM-subjects were 0.0 and 0.85 points. (Data for Part II of the End Test are not reported in tables.)

had been expended by the teachers in these grades to make the arithmetic sensible to their pupils. This fact should prevent unwarranted generalizations, on the hypothesis that the differences obtained for these children are of universal validity and would be secured with any other population as large as that in this study and without regard for arithmetical background. In the numbered conclusions below it may be assumed that each is prefaced by the phrase: "For children with this particular body of arithmetical experience in the preceding two grades."

The second caution arises from the nature of the experimentation. Both the D- and the EA-procedures were taught in two ways, rationally and mechanically. The attempt was made to hold the mechanical instruction to the lowest possible level of meaning and to make the rational instructions as meaningful as possible. A comparison of the two types of instruction (see Appendix C) will reveal the large dissimilarity in teaching which was proposed and which was emphasized in the directions to the teachers. But this dissimilarity, clear enough in theory, almost certainly did not exist so completely in practice. Teachers of DM- and EAM-classes could scarcely teach *less* mechanically than they were asked to, but they could assuredly teach *more* meaningfully than they were supposed to. Similarly, the teachers of DR- and EAR-classes were far more likely to err in the direction of mechanical instruction than in the direction of more meaningful instruction. Hence, to the extent that teachers departed from their assigned programs of instruction, they reduced the gap between these programs; and the experimental differences which were secured are to be regarded as *minimal*. There is a strong probability that the real differences are larger than those reported.<sup>6</sup>

### *Summary with Respect to Procedures*

Chart I has been prepared to facilitate comparisons among the four experimental sections with respect to sixteen functions on which measures were secured. The legend at the bottom of the chart explains the characters which are entered in the various rows.

1. Even a casual inspection of Chart I reveals the impossibility of deciding simply and finally between D and EA as the preferred procedure for teaching borrowing. Yet in the research previously

<sup>6</sup> Facts contained in the teachers' diaries, partly summarized in chap. vi, convincingly support this argument of "minimal differences." More than once evidence was found in the diaries of DM- and EAM-teachers that explanations, supposedly withheld in their classes, were used far beyond the limit set in their instructional directions.

CHART I. CENTER A—SUMMARY OF EXPERIMENTAL COMPARISONS

| Function Measured               | DR vs.<br>DM<br>(1) | DR vs.<br>EAR<br>(2) | DR vs.<br>EAM<br>(3) | DM vs.<br>EAR<br>(4) | DM vs.<br>EAM<br>(5) | EAR vs.<br>EAM<br>(6) |
|---------------------------------|---------------------|----------------------|----------------------|----------------------|----------------------|-----------------------|
| <i>I. Process Taught</i>        |                     |                      |                      |                      |                      |                       |
| A. Accuracy                     |                     |                      |                      |                      |                      |                       |
| 1. End Test, Part I             |                     |                      |                      |                      |                      |                       |
| As a whole. . . . .             | DR                  | dr                   | DR                   | EAR                  | EAM                  | ear                   |
| Borrowing. . . . .              | DR                  | DR                   | DR                   | ear                  | EAM                  | ear                   |
| Nonborrowing*. . . . .          | dr                  | dr                   | dr                   | ear                  | eam                  | ear                   |
| 2. Retention Test               |                     |                      |                      |                      |                      |                       |
| As a whole. . . . .             | DR                  | ear                  | eam                  | EAR                  | EAM                  | ear                   |
| Borrowing. . . . .              | DR                  | ear                  | eam                  | EAR                  | EAM                  | ear                   |
| Nonborrowing. . . . .           | DR                  | ear                  | eam                  | EAR                  | EAM                  | ear                   |
| B. Rate                         |                     |                      |                      |                      |                      |                       |
| 1. End Test, Part I             | dm                  | EAR                  | EAM                  | EAR                  | eam                  | ear                   |
| 2. Retention Test. . . . .      | dr                  | DR                   | eam                  | dm                   | EAM                  | EAM                   |
| C. Smoothness of Performance    |                     |                      |                      |                      |                      |                       |
| 1. Interview I. . . . .         | dm                  | DR                   | EAM                  | DM                   | EAM                  | EAM                   |
| 2. Interview II. . . . .        | dm                  | dr                   | eam                  | DM                   | eam                  | EAM                   |
| D. Degree of Understanding      |                     |                      |                      |                      |                      |                       |
| 1. Interview I. . . . .         | DR                  | dr                   | DR                   | EAR                  | EAM                  | EAR                   |
| 2. Interview II. . . . .        | DR                  | DR                   | DR                   | EAR                  | EAM                  | EAR                   |
| <i>II. Untaught Process</i>     |                     |                      |                      |                      |                      |                       |
| A. Accuracy, End Test, Part III |                     |                      |                      |                      |                      |                       |
| As a whole. . . . .             | DR                  | DR                   | DR                   | ear                  | dm                   | ear                   |
| Borrowing. . . . .              | DR                  | DR                   | DR                   | dm                   | dm                   | ear                   |
| Nonborrowing*. . . . .          | dr                  | ear                  | dr                   | ear                  | eam                  | ear                   |
| B. Rate. . . . .                | dm                  | EAR                  | dr                   | EAR                  | dm                   | EAR                   |

\*C.R.'s not computed since the test included only three or four examples of this type. The letters, where entered, indicate only the *direction* of the advantage, if any.

## EXPLANATION OF SYMBOLS:

dr = difference in favor of DR, but C.R. smaller than 1.5.

dr = difference in favor of DR; C.R. 1.6-2.9.

DR = difference in favor of DR; C.R. 3.0-4.9.

DR = difference in favor of DR; C.R. 5.0, or larger.

done on the problem just such a decision has usually been made in favor of EA. This decision followed from the data collected; but, as the present study shows, these data were incomplete. The criteria employed were those of rate and accuracy of computation in the process taught; but there are other criteria no less important. When these other criteria are taken into consideration, one must give due weight to the kind of teaching which is to be done and to the purposes which are to be achieved. In one set of conditions D (M or R) may be preferable to EA (M or R); under another set of conditions the advantage may lie with EA (M or R).

2. According to the comparisons in column (1), if one is to teach

D at all, one had better do it rationally. Of the sixteen comparisons, twelve favor DR, the differences being reliable in nine instances and nearly so in one more. Of the four differences in favor of DM, none is reliable. Two of these latter differences relate to rate of computation, both of them immediately following the period of instruction. It is usually conceded that rate of work is less important than accuracy of work in the early stages of learning. Moreover, it is to be noted that the slight initial advantage of DM-subjects in rate of work had been lost by the time the Retention Test was given.<sup>7</sup> The other two unreliable differences in favor of DM were in smoothness of performance. It will probably be granted that this advantage is important only when it leads, as it did not here, to superior computation. As a matter of fact, the measures of smoothness of performance were included in the present study as much to isolate this aspect of pupil behavior from measures of understanding as for any other reason.

3. The case for EAR as opposed to EAM is not so clear cut (column 6). Since understanding proved so beneficial in the case of the D-procedure, it was reasonable to expect that it would be equally efficacious in the case of EAR. True, twelve of the sixteen comparisons favor EAR, but only in three instances are the differences reliable. On the other hand, three of the differences favoring EAM are reliable. These last three differences occur in connection with rate of computation (Retention Test only) and smoothness of performance. The three reliable differences favoring EAR relate to rate of work (Transfer Test only) and to degree of understanding. Yet the superiority of the EAR-pupils in understanding availed them relatively little. True, they generally excelled in accuracy in the process taught, but by a very narrow margin. And in the place where greater understanding would seemingly reveal itself most emphatically—the Transfer Test—their margin of advantage remained slight. At a later point in the chapter an attempt will be made to account for the anomalous results obtained by the EAR- and the EAM-subjects.

4. The showing for DM is uniformly poor. Its relative standing

<sup>7</sup> This finding is all the more interesting in view of the fact that, according to their diaries, two DM-teachers disregarded directions and gave much more practice in borrowing than was intended. Practically, this experimental error may be overlooked here and in other comparisons where DM-classes are involved, for even with the extra practice DM-pupils as a whole showed up poorly on the Retention Test.

with DR (column 1) has already been discussed. As compared with EAR (column 4), DM was reliably superior only on smoothness of performance. As compared with EAM (column 5), it has no reliable advantage in any of the sixteen measures. The significance of these findings will be considered later on.

5. In ten comparisons (six reliable) the DR-subjects excel the EAR-subjects; in six comparisons (two reliable) the relationship is reversed. DR produced a greater degree of understanding (the *dr* for Interview I stands for a C.R. of 2.9, which can be accepted as representing reliability), and this greater understanding functioned on the Transfer Test, making for greater accuracy on the test as a whole and on the borrowing examples considered separately. DR's superiority in accuracy in the process taught (End Test) was gone at the time of the Retention Test (though the differences in three comparisons are unreliable). In rate of work the differences are inconsistent, one section having the advantage at one time, the other at another time. Obviously, the choice between the two procedures is a matter of opinion, to be decided primarily by the measures to which weight is attached. Students of arithmetic to whom understanding and transfer value are desiderata will certainly favor DR. Students of the subject to whom these learning products are unimportant might favor EAR.<sup>8</sup>

6. There remains but one set of comparisons, that for EAM with the other procedures. Its advantages over DM (column 5) have already been treated; likewise its uncertain status with respect to EAR (column 6). As compared with DR (column 3), EAM is superior in seven instances, inferior in nine. Six of the nine differences for DR are reliable, and these relate to understanding and to accuracy (End Test as a whole and in the case of borrowing; Transfer Test, the same). It is somewhat surprising that the DR-section did not secure reliably higher accuracy scores (rather than unreliably lower scores) on the Retention Test. An explanation is found in the diary statements of two EAM-teachers that they greatly exceeded the amount of practice they were supposed to assign during the Postexperimental Period. The reliable differences for EAM have to do with rate of work and with smoothness of performance.

<sup>8</sup> One spurious factor, reported in teachers' diaries, worked to the serious disadvantage of DR. This was the excessive number of pupil absences during the period of experimental teaching. The exact effect of this factor cannot of course be assessed quantitatively, but it was of enough consequence to elicit annoyed comments from the teachers concerned.

The relative importance of these points of advantage will vary from interpreter to interpreter. As for the writers, the vote is in favor of DR.

### *Summary with Respect to the Crutches*

The discussion of the usefulness and persistence of the crutches, completed on page 71 above, is so recent as to require no extended statement here. It suffices to say that:

1. The crutch taught in connection with the D-procedure apparently served useful ends.

2. The crutch taught in connection with the EA-procedure did some good, and it seems to have hampered its users not at all.

3. The pupils, during the course of instruction, learned and used the crutches in practically all of their written work. Efforts to teach them away from the crutches were not wholly successful. Immediately after they had been shown shorter ways to borrow and after they had been told not to use the crutches, there was a pronounced drop in frequency of use. The trend away from the crutches, which indeed had started before the end of the instructional period, continued in seven of the eight classes up to the time of the Retention Test. How many children would have continued use of the crutches thereafter—and for how long—is an open question. In an earlier investigation which has been cited, almost all children abandoned their crutch. It is possible that the teachers in the present study did not insist so strongly that the crutches be discarded. Or, as has been pointed out, the children may have had too little practice in the Post-experimental Period to learn to get along without these aids.

### *Interpretation*

1. The present study had its origin in the belief that previous research had oversimplified the issues. In practically every instance (a) kind of teaching (mechanical vs. rational) had been disregarded as a factor; and it may be supposed that the teaching was usually of the kind here described as mechanical. (b) The relative merits of procedures were determined (almost always) solely upon the basis of rate and accuracy of computation in precisely the process taught. But there are other criteria equally important. (c) Age of subjects or, perhaps better, their familiarity with borrowing—these conditioning factors were not adequately considered as affecting the results. In most of the earlier research the subjects were removed by as many as two years from the time of learning to borrow. It

would seem to be important to determine the comparative advantage of borrowing procedures at the time of learning. The importance of each of these factors, it is believed, has been demonstrated, and thus yet another (and more comprehensive) study of the question is justified.

2. There seem to be sound reasons to recommend that borrowing be taught by the procedure here called DR, or by a procedure like it in general pattern and purpose. This proposal, of course, assumes that understanding and transfer value are critical issues.

3. If the sole criteria for evaluating borrowing procedures are to be rate and accuracy of computation *in the process taught*, then a reasonably good case may be made for EAM. This conclusion is in complete agreement with that of other investigators, in whose studies these have been the only criteria, and borrowing has probably been taught mechanically.

4. The failure of EAR to show large and consistent advantages over EAM, analogous to the differences of DR over DM, requires some comment. Various hypotheses suggest themselves. (a) The EA-procedure may be difficult really to rationalize though it is easy to master in a mechanical fashion. True, the EAR-subjects secured higher measures of understanding on the interviews than EAM-subjects, but their superiority could possibly have been spurious. They had been trained in verbal patterns which were designed to embody the rationale of the procedure. The EAM-pupils had not. The EAM-subjects may therefore have suffered disproportionately in explaining their procedure because of their lack of experience in expressing themselves clearly and fully. (b) The EAM-classes may have started the experiment with unnoted advantages. It is a fact that they were brighter than the EAR-children (higher IQ, Table 2), and this advantage may have provided them with special ability to work out the rationale of borrowing for themselves, even though they remained unable to verbalize their explanations satisfactorily.<sup>9</sup> An-

<sup>9</sup> There can, of course, be "understanding" without verbalization, even without the ability to put the "understanding" into words. This condition is well discussed by Gertrude Hendrix, "A New Clue to Transfer of Training," *Elementary School Journal*, XLVIII (1947), 197-208.

In the present investigation the probability of un verbalized understanding is not very large, in spite of the fact that the subjects were dealing with concepts, relationships, etc., which are commonly carried by words. The interviews, planned to secure evidence on understanding, were kept flexible. The interviewers were encouraged to probe for understanding, and were directed to give credit for understanding whenever subjects gave reasonable explanations, no matter how poorly expressed.

other such advantage may have been a better quality of learning the arithmetic of Grades 1 and 2, an advantage which would have had the same effects as their higher average IQ. (c) The EAM-teachers may have departed from their explicit instructions, even quite unwittingly, and supplied explanations which were supposedly denied them. (d) The EAR-teachers may have taught the rationale of borrowing less carefully and less thoroughly than they were expected to.

Of the four tentative explanations, the first seems most reasonable to the authors, namely, that the rationale of borrowing by EA is very difficult to teach, or at least so difficult that it cannot be well done in three weeks. Corroborative data in support of this statement will be presented in Chapter VI, in which information supplied by teachers in their logs will be summarized.

5. Up to this point all intersection comparisons with respect to accuracy have been based upon the reliability of differences in mean performance as measured by Critical Ratios. In other words the sole consideration has been statistical, and no attention has been paid to the educational significance of the differences found.

In Chapter II criticisms were offered of the interpretations given their error data by Johnson (p. 9) and by Murray (p. 14), on the ground that their statistical evidence for EA is educationally insignificant. When Johnson's error data are converted into *accuracy* scores, the largest of his differences amounts to about one subtraction per hundred—a difference certainly too small to warrant a change from D to EA as the method for teaching borrowing. Likewise, Murray's error data, when treated in a similar fashion, show an advantage for EA of only three or four subtractions per hundred. In both instances the "subtractions" referred to are not for examples as wholes but for operations with pairs of numbers (almost the separate subtraction combinations). Since more than one such error could occur in a single example, the numbers given by Johnson and Murray cannot be applied to whole examples, and the per cent differences in accuracy reported above are certainly too large.

In the present study the scores refer not to errors but to accuracy, and the units upon which accuracy scores are based are not separate "subtractions" but examples as wholes. It is only fair then to inquire into the educational significance of the statistical evidence summarized in this chapter. The essential data are tabulated in Table 25. Only the differences found to be reliable are included, for it is these

TABLE 25

CENTER A—EDUCATIONAL SIGNIFICANCE OF DIFFERENCES IN ACCURACY

| Test                                                   | Reliable<br>Differences<br>between                  | Amount of Difference in      |                   |
|--------------------------------------------------------|-----------------------------------------------------|------------------------------|-------------------|
|                                                        |                                                     | Number<br>Examples           | Per Cent          |
| 1. End Test                                            |                                                     |                              |                   |
| As a whole.....<br>(Table 7)<br>(18 examples)          | DR vs. DM<br>DR vs. EAM<br>EAR vs. DM               | 2.60<br>1.10<br>1.95         | 14<br>6<br>11     |
| Borrowing examples.....<br>(Table 8)<br>(14 examples)  | DR vs. DM<br>DR vs. EAR<br>DR vs. EAM<br>EAM vs. DM | 1.67<br>0.85<br>0.75<br>0.92 | 12<br>6<br>5<br>7 |
| 2. Retention Test                                      |                                                     |                              |                   |
| As a whole.....<br>(Table 10)<br>(42 examples)         | DR vs. DM<br>EAR vs. DM<br>EAM vs. DM               | 5.00<br>6.74<br>5.81         | 12<br>16<br>14    |
| Borrowing examples.....<br>(Table 11)<br>(32 examples) | DR vs. DM<br>EAR vs. DM<br>EAM vs. DM               | 4.48<br>5.82<br>4.98         | 14<br>18<br>16    |
| 3. Transfer Test                                       |                                                     |                              |                   |
| As a whole.....<br>(Table 21)<br>(18 examples)         | DR vs. DM<br>DR vs. EAR<br>DR vs. EAM               | 4.85<br>4.54<br>4.93         | 27<br>25<br>27    |
| Borrowing examples.....<br>(Table 22)<br>(15 examples) | DR vs. DM<br>DR vs. EAR<br>DR vs. EAM               | 4.38<br>4.66<br>4.84         | 29<br>31<br>32    |

differences which have been used to evaluate the relative advantages of the four systems of instruction in the experiment.

The table is read as follows: In the End Test as a whole there were eighteen examples, and there were three intersection comparisons which showed reliable differences in accuracy. One of these differences occurred in the case of DR vs. DM, where the mean scores differed by 2.60 examples. This figure, when divided by 18 (the number of examples), shows a difference in accuracy amounting to 14 per cent, or a difference of fourteen correct answers per one hundred examples. In all, the table contains nineteen reliable differences in accuracy, each of which is dealt with precisely as is the difference between DR and DM.

Any decision as to the minimum amount of difference in accuracy that is to be regarded as educationally significant must of course be

a matter of judgment. If the standard is set at five examples per hundred, every one of the nineteen differences is educationally significant. If, however, the standard is set at ten examples per hundred—and this would seem to be too high—then fifteen of the nineteen differences are educationally significant. Even with this last high standard, most of the differences arrived at by purely statistical techniques are seen to mean something for the classroom.

## CHAPTER IV

### THE EXPERIMENT IN CENTER C

The seventeen classes in the Center C experiment had had little pre-third-grade instruction in arithmetic. What they had had consisted largely of abstract drill, with a minimum of attention to meanings and understandings. In this respect these subjects differed considerably from those in Center A (Chapter III). A comparison of the findings for the Center C and the Center A subjects should therefore reveal to some extent the degree to which success in borrowing, whether taught mechanically or meaningfully, is dependent upon the factor of previous arithmetical experience (amount and kind).

That the subjects in Center C were at a disadvantage at the start of the experiment is attested by three facts. (a) Most of the Center C teachers had to take special steps to get their pupils ready. They reported their pupils to be very insecure in their mastery of the subtraction facts, for example, and they instituted daily periods of extra length for intensive practice at this and at other critical points. (b) The start of the experiment proper was deferred for two weeks in Center C, to allow time for the activities mentioned in (a). (Since all the experimental sections in Center C followed the same schedule once the experiment was begun, the postponement in no way invalidates comparisons among them.) (c) In spite of the conditions mentioned in (a) and (b), the Center C pupils were somewhat less proficient on the Pretest than were those in Center A. The means for accuracy for Centers C and A, respectively, were 19.3 and 20.4; for rate of computation the means were 32.2 and 33.0.

#### TREATMENT OF THE EXPERIMENTAL FINDINGS

In every respect the experiments in Centers A and C were parallel. Instruction, varying with the different experimental sections, followed the same pattern; the same tests were administered, and at the same times relative to the experimental schedule; the same kinds of interviews were held; the same sorts of measure were collected; and the same form of statistical treatment was accorded the data. Hence it is possible to report the findings for Center C just about as were those for Center A, but more concisely since the reader of

Chapter III is prepared for the topics to be discussed. Most of the supporting tables are relegated to Appendix A.

## SUBJECTS

Table 26 shows that 136 of the original 613 subjects were lost for experimental purposes, some because of incomplete records and some because of the exigencies of matching groups. The findings, then, are for 477 subjects, with the experimental sections ranging in population from 107 (EAM) to 125 (DR).

TABLE 26  
CENTER C—DISTRIBUTION OF SUBJECTS

| Sections   | Initial<br>Number of<br>Subjects | Subjects Lost         |                        | Number of<br>Subjects<br>Finally Used |
|------------|----------------------------------|-----------------------|------------------------|---------------------------------------|
|            |                                  | Incomplete<br>Records | Process of<br>Equating |                                       |
| DR.....    | 143                              | 18                    | 0                      | 125                                   |
| DM.....    | 159                              | 23                    | 13                     | 123                                   |
| EAR.....   | 174                              | 25                    | 27                     | 122                                   |
| EAM.....   | 137                              | 23                    | 7                      | 107                                   |
| TOTAL..... | 613                              | 89                    | 47                     | 477                                   |

The data on the equivalence of the four sections at the start of the experiment are contained in Tables 28–32 in Appendix A and are summarized in Table 27, which follows. Six comparisons were made for each of the five factors; hence thirty Critical Ratios are reported. Of this number only three C.R.'s are larger than the criterion (3.00) and are to be taken as representing reliable differences. Of the remaining twenty-seven, four are larger than 2.00, the largest being 2.60 and 2.76. Twenty-three of the C.R.'s are less than 2.00, and these differences may safely be regarded as unreliable. The matching was therefore not perfect; but it is probably about as good as could be expected with populations of this size and with so many matching factors.

According to the first line of Table 27, no one of the four sections had a reliably higher average IQ than the other three, the largest C.R. being 2.05 for DR vs. EAM. In the matter of MA, however, two of the C.R.'s are larger than 3.00. DR enjoyed clear advantage in this respect over both EAM and DM. (Experimentally, the difference in favor of DR was of no great consequence. The largest

TABLE 27\*

CENTER C—COMPARABILITY OF EXPERIMENTAL SECTIONS ON INITIAL MEASURES AS REVEALED BY CRITICAL RATIOS

| <i>Factor</i>                | <i>Critical Ratios</i>            |                                    |                                    |                                    |                                    |                                     |
|------------------------------|-----------------------------------|------------------------------------|------------------------------------|------------------------------------|------------------------------------|-------------------------------------|
|                              | <i>DR vs.</i><br><i>DM</i><br>(1) | <i>DR vs.</i><br><i>EAR</i><br>(2) | <i>DR vs.</i><br><i>EAM</i><br>(3) | <i>DM vs.</i><br><i>EAR</i><br>(4) | <i>DM vs.</i><br><i>EAM</i><br>(5) | <i>EAR vs.</i><br><i>EAM</i><br>(6) |
| IQ.....                      | 1.34                              | 1.11                               | 2.05                               | -0.25**                            | 0.20                               | 0.52                                |
| MA.....                      | 3.19                              | 1.72                               | 3.57                               | -1.70                              | 0.39                               | 2.11                                |
| CA.....                      | 1.29                              | 0.39                               | 1.32                               | -0.98                              | 0.10                               | 1.02                                |
| Accuracy of Computation..... | 1.38                              | -0.27                              | 0.67                               | -1.62                              | -0.67                              | 0.92                                |
| Rate of Computation.....     | 4.03                              | 1.17                               | 0.52                               | -2.60                              | -2.76                              | -0.49                               |

\*For the basic tables, see Tables 28-32 in the Appendix.

\*\*Negative Critical Ratios mean that the experimental section mentioned last surpassed that mentioned first.

of the four correlation coefficients for MA and accuracy of computation on Part I of the End Test was .40; and the largest coefficient for MA and rate of computation, same test, was .25.) In CA the four groups may be said to have been equally matched. The same may be said for accuracy of computation on the Pretest, for the largest of the C.R.'s is only 1.62. In rate of computation DR reliably surpassed DM; and EAR surpassed DM, and EAM surpassed DM, by amounts that are not, however, reliable.

If the C.R.'s are compared by columns, it is evident that DR was consistently superior to DM, but reliably so only with respect to MA and to rate of computation. Likewise, DR excelled EAR in four cases, but never reliably. In all measures DR stood higher than EAM, especially in MA. The difference in MA is reliable, but see the note above regarding the experimental significance of this factor. EAR had slight advantages over DM with respect to four factors, and a larger but unreliable advantage with respect to rate of computation. EAM and DM were alike in four factors, but EAM rated somewhat better in rate of computation. The differences between EAM and EAR may be disregarded as unreliable even in the case of MA, where the comparison of means favors EAR.

It may be concluded from these intersection comparisons that no one group had a clear-cut advantage over any other, save possibly DR as compared with DM. Wherever differences exist, however, they must of course be attended to in any conclusions which are drawn.

## RESULTS: PROCESS TAUGHT

*Accuracy of Computation*

By "process taught" is meant the particular skill which was the object of instruction in the experiment proper, namely, borrowing from tens. Measures of proficiency in this skill were secured from Part I of the End Test, in which fourteen of the eighteen examples were of this type, and from the Retention Test, in which thirty-two of the forty-two examples were of this type.

The superiority of meaningful instruction in borrowing by decomposition over mechanical instruction with this procedure is emphatically clear from the data for DR and DM in column (1) of Table 33. Regardless of whether the accuracy measures are for the End Test or for the Retention Test, the DR-subjects outscored the DM-subjects by wide margins. (Since there were only four non-borrowing examples in Part I of the End Test, no C.R.'s were calculated for this type of subtraction.)

TABLE 33

CENTER C—SUMMARY OF RESULTS: ACCURACY IN THE PROCESS TAUGHT\*

| <i>Tests</i>        | <i>Critical Ratios</i>       |                               |                               |                               |                               |                                |
|---------------------|------------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|--------------------------------|
|                     | <i>DR vs.<br/>DM<br/>(1)</i> | <i>DR vs.<br/>EAR<br/>(2)</i> | <i>DR vs.<br/>EAM<br/>(3)</i> | <i>DM vs.<br/>EAR<br/>(4)</i> | <i>DM vs.<br/>EAM<br/>(5)</i> | <i>EAR vs.<br/>EAM<br/>(6)</i> |
| 1. End Test, Part I |                              |                               |                               |                               |                               |                                |
| As a whole.....     | 13.40                        | 4.35                          | 13.61                         | -9.40                         | 1.53                          | 10.04                          |
| Borrowing.....      | 6.33                         | 2.95                          | 10.96                         | -3.36                         | 5.00                          | 8.16                           |
| 2. Retention Test   |                              |                               |                               |                               |                               |                                |
| As a whole.....     | 5.67                         | 3.08                          | 6.31                          | -2.48                         | 1.15                          | 3.56                           |
| Borrowing.....      | 4.04                         | 1.67                          | 5.02                          | -2.17                         | 1.29                          | 3.38                           |
| Nonborrowing.....   | 11.08                        | 8.00                          | 9.64                          | -2.50                         | -0.53                         | 1.82                           |

\*For the basic tables here summarized, see Tables 34-39 in Appendix A.

Much the same kind of statement may be made for the EAR-procedure (column 6): the EAR-subjects were more accurate than the EAM-subjects in all five comparisons, in four by reliable differences. For these particular subjects then (that is, those in Center C) meaningful instruction "paid off" without regard for the particular procedure (D or EA) that was taught.

As between the two meaningfully taught sections (column 2), the advantage is consistently for the DR-subjects, the differences in the

means for accuracy being reliable in three instances, practically so in a fourth (borrowing, End Test), but not so in the case of the borrowing examples on the Retention Test. Here is evidence of the greater ease of learning D initially, but also some evidence that with time the EA-procedure may produce equal accuracy. The greater relative success of the DR-children on the nonborrowing examples of the Retention Test is of interest: apparently the EAR-children at this point in their learning had renewed difficulty in subtracting of the simpler sort.

As between the two mechanically taught sections (column 5), there is no uniform advantage for either procedure; but there is a large difference for DM on the borrowing examples of the End Test. True, the DM-subjects rather consistently, if unreliably, outscored the EAM-subjects; and this fact is at variance with the results of previous research.

In the grosser comparisons, DR with EAM (column 3) and EAR with DM (column 4), the meaningfully taught sections were regularly superior to those taught mechanically. The C.R.'s are reliable in all five comparisons of DR and EAM, and they are reliable in two comparisons of EAR and DM. In the other three comparisons for the two last-named sections the C.R.'s are all larger than 2.00. These results, taken together with others just reviewed, argue that in Center C meaningful teaching was more effective than mechanical teaching in producing accuracy of computation.

There remains the need to determine the effects of learning the new subtraction skill upon the skill previously acquired, that is, of subtracting *without* borrowing. It seems safe to assume from the data in Table 27 that the four sections of Center C were about equally accurate in subtraction of this simpler type at the time when they were introduced to the new skill. Table 40 (the basic tables are omitted) reports what happened to the old skill. The End Test contained four nonborrowing examples. Eighty-eight per cent of the DR-subjects earned scores of 3 or 4 on these examples, as compared with 65 per cent of the DM-subjects, 75 per cent of the EAR-subjects, and 50 per cent of the EAM-subjects. At the other end of the scale, only 4 per cent of the DR-subjects earned scores of 0 or 1, as compared with per cents of 20, 11, and 31 for the DM-, EAR-, and EAM-subjects, respectively. Evidently the new skill interfered with the old skill least for the DR-children, and next in order, for the EAR-, the DM-, and the EAM-children. It should be

inferable that *understanding* of the new operation, especially in the case of the DR-subjects and somewhat less in the case of the EAR-subjects, prevented them from experiencing the losses in the old skill which were suffered by those who were taught mechanically.

TABLE 40

CENTER A—PER CENTS OF SUBJECTS IN EXPERIMENTAL SECTIONS MAKING INDICATED SCORES ON NONBORROWING EXAMPLES

| Test                           | Per Cents of Subjects, by Sections |    |     |     |
|--------------------------------|------------------------------------|----|-----|-----|
|                                | DR                                 | DM | EAR | EAM |
| End Test, Part I (4 examples)  |                                    |    |     |     |
| Scores of 3 or 4 . . . . .     | 88                                 | 65 | 75  | 50  |
| Scores of 1 or 0 . . . . .     | 4                                  | 20 | 11  | 31  |
| Retention Test (10 examples)   |                                    |    |     |     |
| Scores of 8, 9, 10 . . . . .   | 96                                 | 56 | 66  | 65  |
| Scores of 3, 2, 1, 0 . . . . . | 0                                  | 10 | 5   | 9   |

The data in the last two rows of Table 40 are perhaps even more significant. On the Retention Test (ten examples), the DR-subjects maintained or probably improved their already high standard of accuracy, 96 per cent instead of 88 per cent making the best scores, and no children making the low scores of 0, 1, 2, and 3. The EAM-section seems to have improved from the End Test to the Retention Test, but their ratings throughout are low. The records for the DM- and the EAR-sections show, if anything, some deterioration of ability to subtract accurately in nonborrowing examples after the end of the experiment. In conclusion, the D-procedure, when taught with meaning, kept to a minimum interference (retroactive inhibition) from the new to the old skill. The EA-procedure when taught meaningfully had considerably less of this effect. When taught mechanically, on the other hand, the D-procedure seems to have made for a good deal of confusion both on the End Test and on the Retention Test; and the EA-procedure had this effect also, but more so on the End Test than on the Retention Test.

#### *Rate of Computation*

Measures of rate of computation were calculated for the End Test and the Retention Test as wholes. Since in the End Test (that is, Part I) fourteen of the eighteen examples involved borrowing and in the Retention Test thirty-two of the forty-two ex-

amples were of this type, the rate scores may be regarded as indicating predominantly speed of work on the new skill. The relevant data for intersection comparisons are summarized in the first two rows of Table 41.

TABLE 41

CENTER C—SUMMARY OF RESULTS: RATE OF COMPUTATION, SMOOTHNESS OF PERFORMANCE, AND DEGREE OF UNDERSTANDING\*

| <i>Function Measured</i>  | <i>Critical Ratios</i>       |                               |                               |                               |                               |                                |
|---------------------------|------------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|--------------------------------|
|                           | <i>DR vs.<br/>DM<br/>(1)</i> | <i>DR vs.<br/>EAR<br/>(2)</i> | <i>DR vs.<br/>EAM<br/>(3)</i> | <i>DM vs.<br/>EAR<br/>(4)</i> | <i>DM vs.<br/>EAM<br/>(5)</i> | <i>EAR vs.<br/>EAM<br/>(6)</i> |
| Rate of Computation       |                              |                               |                               |                               |                               |                                |
| End Test (Part I)         |                              |                               |                               |                               |                               |                                |
| As a whole . . . . .      | -1.95                        | 1.86                          | 3.13                          | 3.94                          | 5.14                          | 1.29                           |
| Retention Test . . . . .  | 10.29                        | -1.30                         | 5.24                          | -11.30                        | -4.02                         | 6.30                           |
| Smoothness of Performance |                              |                               |                               |                               |                               |                                |
| Interview I . . . . .     | -7.00                        | -2.50                         | -7.83                         | 5.33                          | -0.71                         | -6.17                          |
| Interview II . . . . .    | 2.80                         | 2.60                          | 1.50                          | -0.17                         | -0.83                         | -0.67                          |
| Degree of Understanding   |                              |                               |                               |                               |                               |                                |
| Interview I . . . . .     | 21.50                        | 8.00                          | 20.89                         | -13.50                        | 2.29                          | 15.50                          |
| Interview II . . . . .    | 16.89                        | 3.30                          | 18.89                         | -14.88                        | 2.57                          | 17.13                          |

\*For the basic tables, see Tables 42-45, and 47-48 in Appendix A.

At the end of instruction on the new process (End Test) there was nothing to choose, with respect to rate of work, between DR and DM, between DR and EAR, and between EAR and EAM. DM-subjects computed more rapidly than did EAR- and EAM-subjects, and DR-subjects more rapidly than did EAM-subjects.

Marked changes in relative rate of computation had occurred by the time of the Retention Test. By then the original handicaps imposed upon R-sections—handicaps so far as rate of work is concerned, arising from their use of longer thought processes incident to their working out relationships meaningfully—these handicaps had disappeared, and the DR-section as well as the EAR-section computed more quickly than did the M-sections in every instance. DR-subjects surpassed DM-subjects and EAM-subjects; EAR-subjects surpassed the same two groups of subjects. As between the two R-sections, there was little difference, while the DM-subjects had lost their original advantage over those taught EAM.

It is generally agreed that accuracy of computation should have priority over speed in the early stages of learning. Hence, the

measures of rate on the End Test are less significant than are the corresponding measures on the Retention Test. The facts just reported indicate that, whatever slowness in computation is introduced by meaningful processes at the start of learning, this slowness gives way to computation that is very rapid (as compared with M-procedures) after familiarity has been achieved with those processes. When this faster rate of work is accompanied by superiority as well in accuracy of computation (see Table 33), the argument for meaningful instruction becomes irrefutable.

### *Smoothness of Performance*

At the end of the experimental teaching (Interview I), it is clear (Table 41) that the R-subjects had not yet mastered their thought patterns as completely as had the M-subjects. DM-children were smoother in their performance than were DR- and EAR-children, and EAM-children than were DR- and EAR-children. All the differences are reliable. This finding is not surprising, however inevitable the conditions as reported may be. The thought patterns of the R-children were much more complicated than were those of the M-children. The superiority of the M-sections in smoothness of performance on Interview I sinks into insignificance when an analysis is made of the effects of this superiority: they were less accurate in computation (Table 33), and they understood very much less well the rationale of borrowing (data to be considered below). The processes of the M-sections were smoother merely because they were more glibly employed. (Two M-teachers in Center A spontaneously commented in their diaries on the discrepancy between their pupils' facility in repeating their verbal patterns and their inability to borrow satisfactorily. The same condition in all probability existed in Center C, though the M-teachers volunteered no information on this point.)

As was true in the case of measures of rate of computation, the situation respecting smoothness of performance as exhibited by the four sections was quite different later on, at the time of Interview II. The only relationship to continue undisturbed was that between DM and EAM, where the initial similarity still obtained. DR-subjects were now "smoother" than DM-subjects (the difference is unreliable), and about as "smooth" as the EAM-subjects. And the EAR-subjects had erased their initial disadvantage as compared with the two M-groups. The DR-section, instead of being less smooth in

performance than the EAR-section, was now somewhat better. (The differences in both cases are unreliable.)

Table 46 shows the "adjustments" made by the four sections between nonborrowing and borrowing examples on Interviews I and II. At the end of the experimental teaching the DR- and the EAR-children, who worked somewhat more smoothly on the nonborrowing examples than did the others, made significant changes when they undertook subtraction with borrowing. Their losses in smoothness of performance with the latter type of example amounted respectively to 0.73 and 0.53 scale points. On the other hand, the DM- and the EAM-children dealt with the two types of subtraction in much the same way. By the time of Interview II, the DR-pupils had acquired greater control over their thought patterns for borrowing and employed them about as smoothly as they did the processes they used in simpler subtraction. Similar progress toward smoothness of performance characterizes the record of the EAR-subjects.

TABLE 46  
CENTER C—MEANS FOR SMOOTHNESS OF PERFORMANCE, INTERVIEWS I AND II

| Section  | Means, Interview I |               |            | Means, Interview II |               |            |
|----------|--------------------|---------------|------------|---------------------|---------------|------------|
|          | Borrowing          | Non-borrowing | Difference | Borrowing           | Non-borrowing | Difference |
| DR.....  | 1.91               | 1.19          | 0.73       | 1.24                | 1.15          | 0.09       |
| DM.....  | 1.49               | 1.39          | 0.10       | 1.38                | 1.35          | 0.03       |
| EAR..... | 1.81               | 1.28          | 0.53       | 1.37                | 1.27          | 0.10       |
| EAM..... | 1.44               | 1.44          | 0.00       | 1.33                | 1.27          | 0.06       |

The picture one gets from these comparisons is about this: for the Center C children, M-procedures produced an original superiority in smoothness of performance, but a smoothness which resulted in no great accuracy of work and in no large degree of understanding. Practice on their thought patterns enabled children taught R-procedures to overtake their rivals in smoothness of work, at the same time that they had extra advantages in accuracy and in degree of understanding.

### *Degree of Understanding*

The data for relative degree of understanding are presented in the last two rows of Table 41. As might be expected, the attempt to teach the two procedures D and EA in a meaningful way led to

greater understanding. On Interview I the DR-section was vastly superior in this respect to the DM- and EAM-sections; and in turn the EAR-section, to the same two sections. Of the two procedures, D was clearly the better understood (column 2). On the other hand, when the two procedures were taught mechanically, they produced about the same degree of understanding—which was not much.

The DR-subjects maintained their advantage over the six weeks without instruction (Interview II), showing greater understanding by comfortable margins over DM- and EAM-subjects. The situation for EAR-subjects was similar. The DR-section was still reliably ahead of the EAR-section; and the slight initial advantage of DM over EAM had been maintained.

It is to be remembered that the children in Center C had had little meaningful instruction prior to the start of the experiment. In view of these facts, several considerations are worthy of mention. (a) In spite of their lack of previous meaningful experience these children could, and did, profit from the rationalization of the two subtraction procedures. (b) D was rationalized more readily than was EA. (c) The children who were taught the procedures in a mechanical manner did not discover the underlying relationships through their own efforts. On the contrary, they apparently showed the effects of their earlier instruction and were satisfied to deal with the new skill at the same low level of understanding. Both statements are corroborated in Chapter VI by citations to the teachers' diaries.

#### UNTAUGHT PROCESS: EVIDENCE OF TRANSFER

Part III of the End Test contained eighteen examples. All except three required borrowing, but in a new way. In these fifteen examples the children had to apply borrowing to the figures in the hundreds' column or to the figures in both the hundreds' and the tens' columns. Hence, they worked with an "untaught process," and their relative success reveals the degree to which the various groups of children were able to transfer what they had learned.

#### *Accuracy*

According to the first two rows of data in Table 49 the teachers who tried to invest the D-procedure with meaning produced a type of learning which transferred very well indeed. In every comparison with the DM-and the EAM-subjects, the DR-subjects held the advantage, and by wide margins. The teachers of EAR were not

so successful. Their pupils outscored EAM-children in both comparisons, but did no better than the DM-subjects. Furthermore, they were much inferior in accuracy to the DR-subjects. Here is further evidence that the EA-procedure is more difficult to teach than is the D-procedure; that is to say, more difficult to teach when understanding is regarded as an important learning outcome. That the EA-procedure was also inferior to the D-procedure when both were taught mechanically is indicated by the figures in the next to the last column.

### Rate

The findings for rate of computation are quite different from those for accuracy. The DM-section worked more rapidly than did any other. The comparisons among the three other sections (DR, EAR, and EAM) reveal about the same rate of work. The advantage of the DM-section over the DR-section in rate of work is of no consequence, however, in view of their greater tendency to error; and in general the results for accuracy are of greater significance for educational practice than are those for rate of computation.

TABLE 49  
CENTER C—TRANSFER TEST: RELATIVE ACCURACY AND RATE\*

| Function Measured    | Critical Ratios |               |               |               |               |                |
|----------------------|-----------------|---------------|---------------|---------------|---------------|----------------|
|                      | DR vs.<br>DM    | DR vs.<br>EAR | DR vs.<br>EAM | DM vs.<br>EAR | DM vs.<br>EAM | EAR vs.<br>EAM |
| Accuracy             |                 |               |               |               |               |                |
| End Test (Part III)  |                 |               |               |               |               |                |
| As a whole . . . . . | 8.29            | 8.73          | 15.77         | 0.17          | 7.48          | 7.57           |
| Borrowing . . . . .  | 6.47            | 7.37          | 14.03         | 0.19          | 7.35          | 8.00           |
| Rate                 |                 |               |               |               |               |                |
| End Test (Part III)  |                 |               |               |               |               |                |
| As a whole . . . . . | -4.22           | 1.62          | 0.75          | 5.79          | 4.54          | -0.72          |

\*For the basic tables, consult Tables 50-51 and 53 in Appendix A.

### USEFULNESS AND PERSISTENCE OF THE CRUTCHES

As was explained in Chapter III, estimates of the value of the crutches taught the DR- and the EAR-sections can only be inferred from such data as have been presented. This is true for the reason that the crutches were taught as inseparable parts of the instructional programs followed in these two sections. On the whole the DR- and the EAR-pupils outscored the DM- and the EAM-pupils on the

more significant measures. (The findings which support this statement have been presented too recently to warrant a rehearsal in this place.) From this fact it may be deduced that if the crutches were not helpful, they most certainly were not harmful; and it is not unreasonable to deduce that they actually aided their users in learning the new subtraction process. Chapter VI, which summarizes the reports of the experimental teachers, will show that this inference is sound.

Data relating to the persistence of the crutches in Center C were collected as in Center A. The basic tables are omitted here as in the case of Center A. Instead, the relationships are presented in curves in Figure 3 and Figure 4, constructed as were Figures 1 and 2 for Center A.

Extent of use of the crutches was calculated for Figure 3 (a) by noting on five work sheets the total number of times the crutches were used and (b) by dividing this total by the possible number of

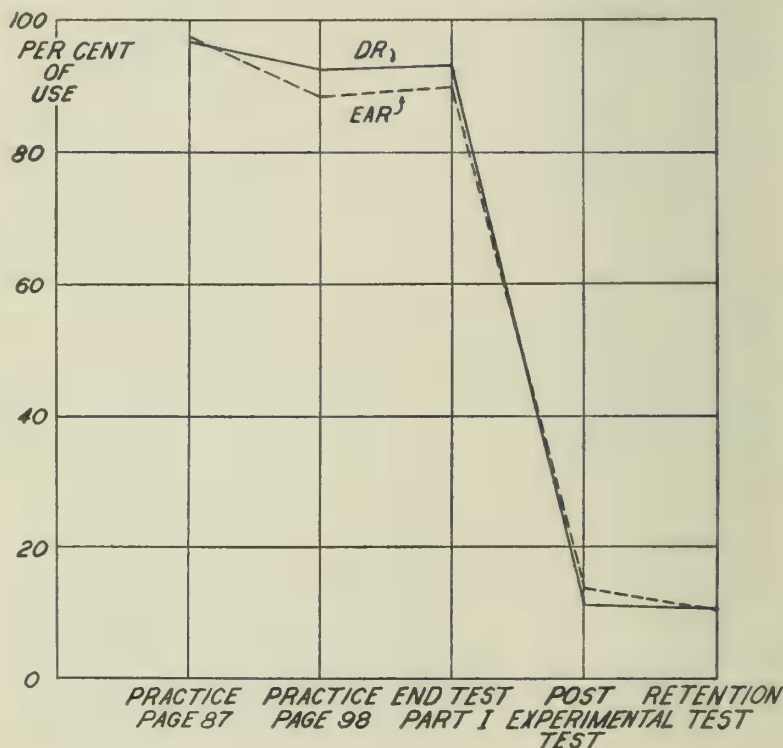


FIG. 3. Actual number of crutch uses divided by possible number of uses, at successive times of measurement.

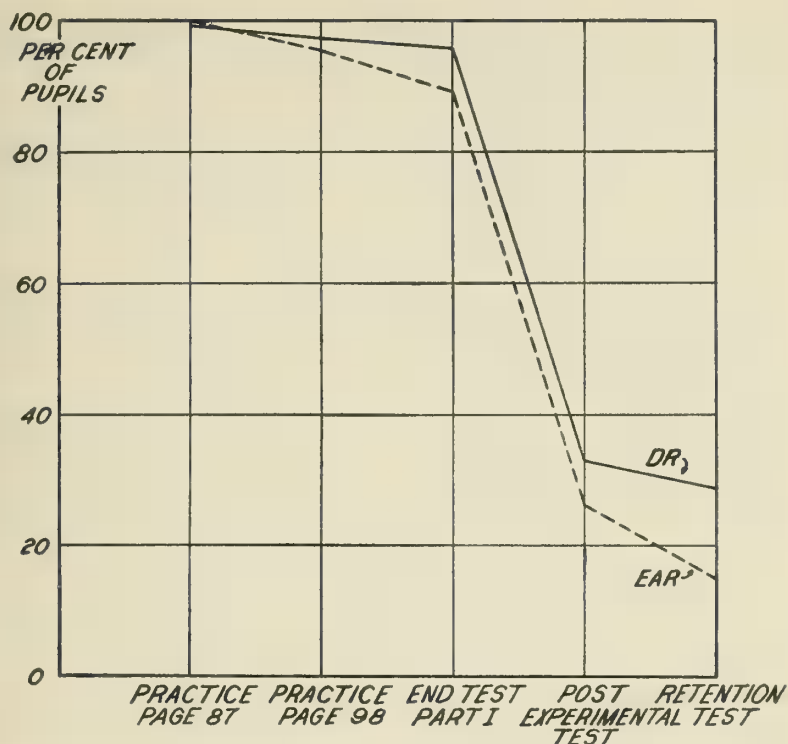


FIG. 4. Per cents of pupils using crutches, at successive times of measurement.

times when they could have been employed. Thus, the statistic 96.0 per cent means that a crutch was used in 96 out of every possible 100 chances. The curves for the two crutches are very similar. In both instances they were employed throughout the experimental teaching 88 per cent or more of the time. With the specific direction to eliminate the crutches after the Retention Test, use fell to less than 15 per cent on the Postexperimental Test. There was no further decrease in reliance on the device in the case of the DR-crutch; but the decrease continued slowly during the ensuing weeks in the case of the EAR-crutch.

The measures upon which the curves in Figure 4 are based are for users rather than for uses of crutches. If, at any time observations were made, a given child used a crutch even once in his work, he was counted as a crutch user for the purpose of Figure 4. Thus, on Practice Page 87 every child in the EA-classes and 99.2 per cent of

those in D-classes used their crutches at least once. Eighty-nine per cent or more of the children employed their devices throughout the period of instruction. After the End Test the per cents dropped drastically to 33 (DR) and to 24 (EAR), with the further losses thereafter particularly in the case of the EAR-subjects. How much longer these children would have used the crutches, and what steps might have been taken to discourage use (if indeed it should have been discouraged)<sup>1</sup> is not known.

The curves for Center A (Figures 1 and 2, pp. 69 and 70) are remarkably like those just presented for Center C. In both centers use of the crutches was general throughout the period of teaching, as it was supposed to be; and in both centers the introduction of the shorter algorisms after the End Test brought large-scale reductions in use, particularly so in Center C. But in Center A the D-crutch seemed to be somewhat easier to eliminate than the EAR-crutch, whereas in Center C the relationship seems to have been the reverse.

#### SUMMARY ON TEACHING

Chart II assembles the findings for Center C save for data on the usefulness and the persistence of the crutches. As in the case of Chart I for Center A (p. 73) all figures are omitted, and the comparisons among the experimental sections are expressed in terms of Critical Ratios. The general tenor of the results for Center C corroborates that for Center A extraordinarily well.

1. If the D-procedure is to be taught, it had best be taught meaningfully. In the sixteen comparisons between DR and DM (column 1) DR was ahead in thirteen. DM excelled in rate of computation in two cases (one reliable), but its inferiority at other more significant points negates this advantage. It must be admitted that at the start of the experiment the DR-sections rated higher in MA (Table 29) and in speed of computation (Table 32); but this initial superiority hardly explains the differences in the records of the two sections later on. In the first place, DR's advantage in MA has already been discounted (pp. 82-83) by data which show MA not to have been very closely related to success in borrowing. In the second

<sup>1</sup> As noted earlier, Wilburn studied persistence of the D-crutch (D. Banks Wilburn, "Learning to Use a Ten in Subtraction," *Elementary School Journal*, XLVII [1947], 461-466). In private correspondence with the senior author of this monograph Professor Wilburn argues convincingly that children should not be hurried away from crutch use, and that, rather, they should be allowed to use the device until they can themselves move to a higher, more abstract level of performance in which they do not need it.

CHART II. CENTER C—SUMMARY OF EXPERIMENTAL COMPARISONS

| Function Measured                   | DR vs.<br>DM<br>(1) | DR vs.<br>EAR<br>(2) | DR vs.<br>EAM<br>(3) | DM vs.<br>EAR<br>(4) | DM vs.<br>EAM<br>(5) | EAR vs.<br>EAM<br>(6) |
|-------------------------------------|---------------------|----------------------|----------------------|----------------------|----------------------|-----------------------|
| <i>I. Process Taught</i>            |                     |                      |                      |                      |                      |                       |
| A. Accuracy                         |                     |                      |                      |                      |                      |                       |
| 1. End Test, Part I                 |                     |                      |                      |                      |                      |                       |
| As a whole.....                     | DR                  | DR                   | DR                   | EAR                  | dm                   | EAR                   |
| Borrowing.....                      | DR                  | dr                   | DR                   | EAR                  | DM                   | EAR                   |
| Nonborrowing..                      | dr                  | dr                   | dr                   | ear                  | dm                   | ear                   |
| 2. Retention Test                   |                     |                      |                      |                      |                      |                       |
| As a whole.....                     | DR                  | DR                   | DR                   | ear                  | dm                   | EAR                   |
| Borrowing.....                      | DR                  | dr                   | DR                   | ear                  | dm                   | EAR                   |
| Nonborrowing..                      | DR                  | DR                   | DR                   | ear                  | eam                  | ear                   |
| B. Rate                             |                     |                      |                      |                      |                      |                       |
| 1. End Test, Part I                 | dm                  | dr                   | DR                   | DM                   | DM                   | ear                   |
| 2. Retention Test..                 | DR                  | ear                  | DR                   | EAR                  | EAM                  | EAR                   |
| C. Smoothness of<br>Performance     |                     |                      |                      |                      |                      |                       |
| 1. Interview I....                  | DM                  | ear                  | EAM                  | DM                   | eam                  | EAM                   |
| 2. Interview II....                 | dr                  | dr                   | dr                   | ear                  | eam                  | eam                   |
| D. Degree of Under-<br>standing     |                     |                      |                      |                      |                      |                       |
| 1. Interview I....                  | DR                  | DR                   | DR                   | EAR                  | dm                   | EAR                   |
| 2. Interview II....                 | DR                  | DR                   | DR                   | EAR                  | dm                   | EAR                   |
| <i>II. Untaught Process</i>         |                     |                      |                      |                      |                      |                       |
| A. Accuracy, End Test,<br>Part III  |                     |                      |                      |                      |                      |                       |
| As a whole.....                     | DR                  | DR                   | DR                   | dm                   | DM                   | EAR                   |
| Borrowing.....                      | DR                  | DR                   | DR                   | dm                   | DM                   | EAR                   |
| Nonborrowing*..                     | dr                  | dr                   | dr                   | dm                   | dm                   | ear                   |
| B. Rate, End Test,<br>Part III..... | DM                  | dr                   | dr                   | DM                   | DM                   | eam                   |

\*C.R.'s not computed, since the test included only three or four examples of this type. The letters indicate the direction of the advantage only.

## EXPLANATION OF SYMBOLS:

- dr = difference in favor of DR, but C.R. smaller than 1.5.  
 dr = difference in favor of DR; C.R. 1.6—2.9.  
 DR = difference in favor of DR; C.R. 3.0—4.9.  
 DR = difference in favor of DR; C.R. 5.0, or larger.

place, the original difference in rate, in favor of DR, was wiped out in the course of the experiment. It seems therefore only reasonable to assume that children learn D better when it is taught rationally.

2. If EA is taught, it too had best be taught meaningfully. In practically all significant ways (column 6), the EAR-children surpassed the EAM-children.

3. The last statement would seem to indicate that the EA-procedure may be rationalized with full benefits to children with limited arithmetical backgrounds. But this inference loses weight when the records of the EAR-subjects are compared with those of the DR-

subjects. According to column 2, the DR-section outscored the EAR-section in fourteen cases, seven times by reliable amounts. The EAR-section was superior by insignificant amounts in rate on the Retention Test and in smoothness on Interview I. Throughout the chapter the superior showing of the DR-children has been repeatedly noted.

4. The meaningfully taught sections rather consistently excelled the mechanically taught sections. (Columns 1 and 6 have already been discussed, but DR-subjects surpassed EAM-subjects as well as DM-subjects as shown in column 3, while the EAR-subjects surpassed the EAM-subjects as well as the DM-subjects, as shown in column 4.)

5. There is some evidence that rational instruction produced greater uniformity in accuracy of computation than did mechanical instruction. On the Pretest (Table 31) the S.D.'s dist of the R-sections (DR, 6.45; EAR, 6.90) were larger than the corresponding measures of variability for the M-sections (DM, 6.30; EAM, 6.38). Yet on all three measures of accuracy on the Retention Test and on two of the three measures of the Transfer Test, the S.D.'s dist of the R-sections were smaller. This fact seems to signify that rationalization had the effect of producing greater homogeneity in learning to borrow.

6. If the D- and the EA-procedures are to be taught mechanically (column 5), there is not a great deal to choose between them. If anything, the record of the DM-section is somewhat better than that of the EAM-children, and in this respect the data for Center C are in disagreement with data reported in other studies.

7. The findings in Center A were generally in favor of DR, but it was clear that a decision as to the subtraction procedure to teach was contingent upon the weights attached to various learning outcomes. In Center C, on the other hand, the decision is less influenced by such considerations. In this latter center DR would be preferred to any other teaching program almost regardless of the learning outcomes to which value may be attached. An explanation for the differences at this point may well lie in the dissimilarity of previous experience in arithmetic. The children in Center A, with their greater understanding of arithmetical processes at the start of the experiment, may have been able to profit therefrom even under mechanical instruction. That this hypothesis has a foundation in fact will be shown in Chapter VI. The children in R-sections in

Center C, on the other hand, had only the meaningful instruction of the experimental period to help them. That they took advantage of this opportunity is attested by the comparisons in Chart II.

8. It is clear that smoothness of performance is a poor criterion of learning. The mechanically taught sections had of course an initial superiority at this point, but this superiority was merely in glibness of pattern, and it was soon lost. The meaningfully taught sections were somewhat "rough" in their processes at the outset, as might be expected from their more involved language patterns; but in time they overcame their initial inferiority. The educational implications of this finding should be clear to the classroom teacher.

9. Rate of computation is also, by itself, an inadequate criterion of learning, particularly when it is accompanied by a tendency to error and by absence of understanding.

10. "Understanding" is of course a relative term. The EAR-children, for example, manifested much more understanding of borrowing than did the DM-children at the times when they were interviewed. Yet the understanding of the EAR-children was relatively limited, for in accuracy on the Transfer Test they showed no superiority over the DM-children. Apparently the EAR-children understood fairly well just what they were taught, but they could not carry over this understanding to new types of borrowing.

11. Most students of arithmetic now advocate meaningful instruction in arithmetic (though they do not always agree on details). They admit that extra time may be required for such instruction in the early phases of learning and that accordingly the pace of instruction at first may have to be relatively slow. But they insist that, even so, time is not actually lost; for, once children understand what they are to learn, the pace of instruction may be speeded up in later phases of learning. Moreover, they insist that the effects of meaningful instruction are cumulative. Understanding is said to facilitate learning at later points. Support for these claims is to be found in the record of the DR-section which, unlike the EAR-section, really saw sense in borrowing from tens. Because of their relatively large success on the Transfer Test it can be inferred that the children in this section would encounter little difficulty in extending their subtraction skills into more complicated types of borrowing. On the other hand, the DM- and the EAM-children, as well as the EAR-children, would need considerable attention in mastering these types of computation.

12. There is inferential evidence that the crutches taught the DR- and the EAR-sections helped rather than hindered the learning of borrowing.

13. The crutches were pretty well abandoned after shorter algorithms were put into use, but there was some persistence. The seriousness of this amount of persistence is conjectural; but to the authors it appears to be of relatively little consequence.

#### INTERPRETATION

Among the findings of special educational significance are the four following.

1. Decomposition can be successfully rationalized for children with exceedingly limited backgrounds in meaningful arithmetic (finding 1). A comparison of "degree of understanding" for DR-sections in Center C and in Center A (relatively rich arithmetical backgrounds) is illuminating. The Center A mean on Interview I was 2.14; on Interview II, 1.91. The corresponding means for Center C were 1.95 and 2.11 (low scores representing superior performances). That the *functional value* of the understanding developed in Center C was somewhat less than that in Center A is shown by the comparative scores of the two DR-sections on the borrowing examples of the Transfer Test: Center A, 7.49; Center C, 6.83. Still, the record of the DR-subjects in Center C is impressive and warrants the statement at the beginning of this paragraph.

2. EA cannot be so successfully rationalized in the case of children like those in Center C (finding 2). In this center the DR-means for "degree of understanding" were 1.95 on Interview I and 2.11 on Interview II. For the EAR-sections the two means were 2.59 and 2.44. A comparison of the DR- and the EAR-sections on accuracy with the borrowing examples of the Transfer Test reveals a large superiority for DR (means: DR, 6.83; EAR, 4.84).

3. When both EA and D are taught rationally to children with limited arithmetical backgrounds, the DR-classes outscore the EAR-classes (finding 3). Similar results, differing chiefly in degree, were obtained in Center A. The inference then is that DR is more effective than EAR regardless of the amount and kind of previous arithmetical experience.

4. When EA and D were both taught mechanically in Center C, the D-sections did somewhat better than the EA-sections (finding 4). This result is surprising in the light both of previous research and

of the results obtained in Center A. No explanation for this reversal of advantage is to be found either in the quantitative data or in the teachers' logs.

To these four conclusions of special educational significance is to be added a fifth, namely, the answer to the question: Do the statistical findings respecting accuracy in the intersection comparisons mean anything educationally?

5. A table similar to Table 25 (page 79) for Center A was prepared for Center C. The differences in means for accuracy in all comparisons known to be reliable were found and were converted into percentages. Thirty-one reliable differences were treated in this way, one difference with a C.R. of 2.95 being included. The following tabulation shows the findings:

| <i>Differences in<br/>per cent of</i> | <i>Frequency</i> |
|---------------------------------------|------------------|
| 4 points or less .....                | 2                |
| 5- 9 points .....                     | 12               |
| 10-14 points .....                    | 6                |
| 15-19 points .....                    | 7                |
| 20 points or more .....               | 4                |

If a difference of five correct answers per one hundred subtractions is taken to mean educational significance, then twenty-nine of the thirty-one differences meet this standard. If the criterion is put at ten correct computations per one hundred, then seventeen are educationally significant. The lower standard would seem to be more justifiable.

The relative frequency of the per cent differences of various sizes varied a good deal with the functions measured. On the End Test as a whole, all five per cent differences were larger than 10, ranging from 13 to 21. On the fourteen borrowing examples in this test, the per cent differences were much smaller, only one being larger than 10, with three of 9, one of 5, and one of 4 (for the C.R. of 2.95). Seven of the ten per cent differences on the Retention Test were smaller than 10, there being one of 9, one of 8, one of 7, two of 6, and one each of 5 and 4. On the Transfer Test as a whole, three of the per cent differences amounted to 10 or more, with two of 7. On the fourteen borrowing examples of the same test, all five per cent differences were larger than 10, ranging from 14 to 29.

## CHAPTER V

### THE EXPERIMENT IN CENTER B

From the standpoint of the *total amount* of previous instruction in arithmetic, the pupils in Center B started the experiment with a distinct advantage over the pupils in the other two centers. That is to say, in Center B the arithmetic program as a whole for Grades 1 and 2 was much heavier than in Centers A and C. The results are evident in the relative standing of the three centers in accuracy of computation on the subtraction examples of the Pretest. The median accuracy score for the four sections of Center B was 21.6, with a range of 19.8 to 23.1. For Center A, the median was 20.6, with a range of 19.5 to 21.2; for Center C, the median was 19.4, with a range of 18.6 to 19.9.<sup>1</sup>

On the other hand, from the standpoint of *amount of meaningfully taught arithmetic* they had had in Grades 1 and 2, the subjects in Center B occupied a position intermediate between the positions of Centers A and C. This does not mean that all children in Center B had had the same intermediate amount of meaningful arithmetic. On the contrary, there were wide differences among the classes in this respect. To judge from the teachers' diaries, some classes had had as much meaningful arithmetic as the classes in Center A, while others had had as little as the classes in Center C. It is only by "averaging" these differences in pre-third-grade contacts with meaningful arithmetic that one may assign Center B the intermediate position referred to.

It was impossible in Center B to maintain the same kind of supervision which was provided in the other two centers. That the teachers in this center tended to "get out of line" with regard to directions for teaching, more so than in the other two centers, is evidenced by comments in the teachers' diaries, some of which are reported in Chapter VI. Nevertheless, in spite of these deviations from the plans for instruction the results obtained in Center B agree very closely with those in Centers A and C, a fact which lends con-

<sup>1</sup> It is of interest to note that Center B's initial advantage in nonborrowing subtraction was of little effect when borrowing was introduced. The records made by the experimental sections in this center on the four computation tests given were little, if any, better than those earned in Centers A and C.

siderable support to the reliability of the conclusions to be drawn for the investigation as a whole.

The data for Center B are incomplete at two points. (a) For administrative reasons it was impossible to give the second interviews. Hence, measures for "smoothness of performance" and "degree of understanding" are lacking for the end of the Postexperimental Period. (b) Too, through some error one set of pupils' work papers during the Experimental Period was not received at Duke University. On this account the measures for crutch use at this point are missing for the pupils concerned (the DR-classes).

#### SUBJECTS

The experiment began with a total of 391 pupils. For various reasons this total was reduced to 287, as shown in Table 54. The four experimental sections each comprised slightly more than seventy subjects.

TABLE 54  
CENTER B—DISTRIBUTION OF SUBJECTS

| Sections   | Initial<br>Number of<br>Subjects | Subjects Lost         |                        | Number of<br>Subjects<br>Finally Used |
|------------|----------------------------------|-----------------------|------------------------|---------------------------------------|
|            |                                  | Incomplete<br>Records | Process of<br>Equating |                                       |
| DR.....    | 94                               | 23                    | 0                      | 71                                    |
| DM.....    | 103                              | 23                    | 8                      | 72                                    |
| EAR.....   | 91                               | 17                    | 3                      | 71                                    |
| EAM.....   | 103                              | 21                    | 9                      | 73                                    |
| TOTAL..... | 391                              | 84                    | 20                     | 287                                   |

#### COMPARABILITY OF EXPERIMENTAL SECTIONS

After the sections had been matched with regard to five factors (IQ, MA, CA, accuracy of computation, and rate of computation on the Pretest), they were fairly comparable. In only four of the thirty comparisons (Table 55) are differences between pairs of sections reliable, though in four other instances the C.R.'s equal or exceed 2.60.

Sections DR and DM were well matched on four bases, with section DR having a slight but unreliable advantage in MA. Sections DR and EAR were also well matched in four of the factors, the EAR-children being somewhat (but unreliably) faster in com-

TABLE 55\*

CENTER A—COMPARABILITY OF EXPERIMENTAL SECTIONS ON INITIAL MEASURES AS REVEALED BY CRITICAL RATIOS

| Factor                       | Critical Ratios |               |               |               |               |                |
|------------------------------|-----------------|---------------|---------------|---------------|---------------|----------------|
|                              | DR vs.<br>DM    | DR vs.<br>EAR | DR vs.<br>EAM | DM vs.<br>EAR | DM vs.<br>EAM | EAR vs.<br>EAM |
| IQ.....                      | 0.58            | -0.19**       | -1.83         | -0.87         | -3.08         | -1.80          |
| MA.....                      | 2.40            | 0.28          | 0.23          | -1.97         | -1.97         | -0.04          |
| CA.....                      | 1.93            | 1.46          | 3.57          | -0.53         | 2.46          | 2.71           |
| Accuracy of Computation..... | -1.17           | -2.08         | 0.89          | -0.47         | 2.86          | 3.24           |
| Rate of Computation.....     | -0.89           | -2.91         | 1.77          | -1.90         | 2.60          | 4.68           |

\*For basic tables, See Tables 56-60 in Appendix A.

\*\*Negative Critical Ratios mean that the experimental section mentioned last surpassed that mentioned first.

putation. Sections DR and EAM were about alike in four factors. The children in the former section were, however, generally older. Sections DM and EAR differed little if any in the five comparisons, though EAR held a slight and consistent advantage over DM. Sections DM and EAM were not so well matched as a pair. The difference in IQ, a reliable one, favored the EAM section; the DM section was older and more accurate and quicker in computation, though none of the last-named differences is reliable. Section EAR computed more accurately and more rapidly than Section EAM, the differences being reliable. The advantage in the other three factors, all unreliable, was divided between the two sections.

In general, the matching was adequate for four pairs of sections: DR vs. DM, DR vs. EAR, DR vs. EAM, and DM vs. EAR. It was not so satisfactory in the case of DM vs. EAM. While only one C.R. meets the criterion, 3.00, three others are larger than 2.40. Yet, it cannot be said that *all* the advantage lay with either section. The matching for the sixth pair, EAR vs. EAM, was the least satisfactory; and EAR started the experiment under more favorable conditions somewhat generally than did EAM. (The data summarized in Table 55 will be found in Tables 56 to 60 in Appendix A.)

#### EXPERIMENTAL RESULTS: COMPUTATION

The results of the experiment appear in Chart III, which is similar in form to Charts I and II for Centers A and C, respectively; that is, similar save for two entries. Since the second interviews were not given in Center B, only one row of entries, rather than two rows, occurs after "C. Smoothness of Performance" and after "D. Degree of Understanding."

CHART III. CENTER B: SUMMARY OF EXPERIMENTAL COMPARISONS\*

| Function Measured                        | DR vs.<br>DM<br>(1) | DR vs.<br>EAR<br>(2) | DR vs.<br>EAM<br>(3) | DM vs.<br>EAR<br>(4) | DM vs.<br>EAM<br>(5) | EAR vs.<br>EAM<br>(6) |
|------------------------------------------|---------------------|----------------------|----------------------|----------------------|----------------------|-----------------------|
| <i>I. Process Taught</i>                 |                     |                      |                      |                      |                      |                       |
| A. Accuracy                              |                     |                      |                      |                      |                      |                       |
| 1. End Test, Part I                      |                     |                      |                      |                      |                      |                       |
| As a whole . . . .                       | DR                  | DR                   | dr                   | dm                   | EAM                  | EAM                   |
| Borrowing . . . .                        | DR                  | DR                   | dr                   | dm                   | eam                  | EAM                   |
| Nonborrowing**                           | dr                  | dr                   | dr                   | ear                  | dm                   | ear                   |
| 2. Retention Test                        |                     |                      |                      |                      |                      |                       |
| As a whole . . . .                       | DR                  | DR                   | DR                   | ear                  | eam                  | eam                   |
| Borrowing . . . .                        | DR                  | DR                   | DR                   | ear                  | EAM                  | eam                   |
| Nonborrowing..                           | DR                  | DR                   | DR                   | ear                  | dm                   | ear                   |
| B. Rate                                  |                     |                      |                      |                      |                      |                       |
| 1. End Test, Part I                      | dm                  | dr                   | eam                  | dm                   | eam                  | EAM                   |
| 2. Retention Test..                      | DR                  | DR                   | dr                   | ear                  | eam                  | eam                   |
| C. Smoothness of<br>Performance          |                     |                      |                      |                      |                      |                       |
| 1. Interview I . . . .                   | dr                  | DR                   | eam                  | DM                   | eam                  | EAM                   |
| D. Degree of Under-<br>standing          |                     |                      |                      |                      |                      |                       |
| 1. Interview I . . . .                   | DR                  | ear                  | DR                   | EAR                  | EAM                  | EAR                   |
| <i>II Untaught Process</i>               |                     |                      |                      |                      |                      |                       |
| A. Accuracy, End Test,<br>Part III       |                     |                      |                      |                      |                      |                       |
| As a whole . . . .                       | DR                  | DR                   | DR                   | dm                   | eam                  | eam                   |
| Borrowing . . . .                        | DR                  | DR                   | DR                   | DM                   | eam                  | EAM                   |
| Nonborrowing**                           | dr                  | dr                   | dr                   | ear                  | dm                   | ear                   |
| B. Rate, End Test,<br>Part III . . . . . | DR                  | dr                   | eam                  | ear                  | EAM                  | eam                   |

\*For the basic tables consult Tables 61-75 in Appendix A.

\*\*C.R.'s not computed since the test included only three or four examples of this type. The letters in the chart indicate the direction of the advantage only.

## EXPLANATION OF SYMBOLS:

dr = difference in favor of DR; but C.R. smaller than 1.5.

dr = difference in favor of DR; C.R. 1.6-2.9.

DR = difference in favor of DR; C.R. 3.0-4.9.

DR = difference in favor of DR; C.R. 5.0, or larger.

*DR vs. DM.* According to the entries in column 1 decomposition was much more effectively learned when taught rationally than when taught mechanically. In thirteen of the fourteen comparisons relating in one way or another to computation, the DR-subjects outscored the DM-subjects. Ten of the differences are reliable. Two other entries are dr, for accuracy on nonborrowing examples in Parts I and III of the End Test. These two differences could also be reliable, but no C.R.'s were calculated because of the small numbers of examples of this type in the tests in question. There is one entry of "dr," signifying a pronounced but unreliable difference. It appears in connection with smoothness of performance on the End

Test. That the DR-subjects were superior at this point, even if not reliably so, is rather surprising. The DR-children had to employ much more complicated language patterns in reporting their activities in subtracting. Even so, their performances were "smoother" than those of the DM-children. This last comparison evidences, perhaps even more than do some of the comparisons based upon reliable differences, the extraordinary difficulty the DM-subjects encountered in learning to borrow without rational explanations, difficulty which the DR-subjects escaped because of the way in which they were taught.

*DR vs. EAR.* The superiority of DR over EAR, convincingly clear in Center C and somewhat less so in Center A, is confirmed by the data for Center B (column 2). Thirteen comparisons favor DR, in nine instances by reliable amounts. The only unexpected result is that the EAR-subjects seem to have understood their procedure better than the DR-subjects understood theirs. Yet, this greater degree of understanding (unreliable here)<sup>2</sup> did not function in any of the tests to yield higher scores either in rate or in accuracy of computation.

*DR vs. EAM.* On most of the criteria the record favors the DR-section over the EAM-section (column 3). The reliable differences occur in connection with: accuracy on the Retention Test (all three measures), degree of understanding (one measure), and accuracy on the Transfer Test (two of three measures). On the test given at the end of the Experimental Period and covering precisely the skills taught, the DR-subjects were more accurate also, but not reliably so. As was found in Centers A and C, the EAM-children were perhaps a bit more glib in the use of their language pattern (smoothness of performance) and in two of three instances were quicker in computation. These latter differences are, however, unreliable and may therefore be disregarded in arriving at a final evaluation respecting the merits of DR and EAM. On the record it would have to be said that the DR-classes in Center B mastered their new skill to a greater degree of success than did the EAM-

<sup>2</sup> As a matter of fact, the Center B comparisons involving interview data can be made none too confidently. The distributions of scores, both for smoothness of performance and for degree of understanding, were severely skewed. The reason is unknown. Whatever the reason, the resulting lack of variability adversely affected the data for the purposes of correlation as required in the Shen formula. Some other form of statistical treatment might have been substituted, but with no great certainty of uniformity of interpretation.

classes; and this finding is in agreement with previous findings in this study.

*DM vs. EAR.* If the data in column 4 establish anything, they establish the fact that EA as it was taught in Center B was extremely difficult for the EAR-subjects to master. In only one comparison, degree of understanding, is there a reliable difference in favor of EA over D, and certainly any other result would have been most unexpected. After all, the teachers of EAR attempted to develop understanding, and those of DM were directed to avoid doing so. Yet the understanding attained by the EAR-subjects seems to have been rather superficial, for it did not produce reliably greater accuracy in any of the nine possible instances. Indeed, in the very place where understanding had its best chance to function, namely, in the new types of borrowing in the Transfer Test, the advantage, and a reliable advantage at that, lay with the DM-subjects.

*DM vs. EAM.* The EAM-classes were consistently ahead of the DM-classes (eleven of the fourteen comparisons), but the differences are reliable in only four instances (column 5). The conclusion seems to be that, if both procedures are taught mechanically, EA produces better results, a conclusion which agrees with that of previous research. In the case of Center B, however, the advantage for EA may be somewhat spurious, for it is known that some of the EAM-teachers disregarded the experimental directions and taught arithmetic for periods unconscionably long. (See Chapter VI.)

*EAR vs. EAM.* In ten of the fourteen comparisons (column 6) the EAM-children excelled the EAR-children, five times by reliable amounts. The showing of the EAR-subjects in Center B is poor indeed. So much is manifest in the data of Chart III. But why? Why did these children profit to no greater extent from rational instruction? True, on the criterion of degree of understanding they were reliably superior (but see p. 104, n. 2), and this superiority was expected. But their anticipated advantage at this point helped them very little where it should have helped most, namely, in the new-type subtraction examples of the Transfer Test. Moreover, as has been shown in Table 55, the EAR-section began the experiment with real advantages over the EAM-section. One explanation for the relative failure of the EAR-section is to be found in the fact noted above that certain EAM-teachers exceeded their instructions and spent undue amounts of time on daily lessons. Even so, they could scarcely have gone far enough in this deviation from the experimental plan

to account for the general inferiority of the EAR-children. One is left with the conviction that in Center B the teachers of EAR-classes found it impossible to invest the process of borrowing with any effective degree of understanding.

#### EXPERIMENTAL RESULTS: USEFULNESS AND PERSISTENCE OF THE CRUTCHES

The data pertaining to persistence of the crutches are incomplete at one point. The teachers of DR-classes failed to supply their pupils' lesson sheets for Practice Page 98. This page was assigned for a lesson about two thirds of the way through the period of experimental teaching. In both Center A and Center C the extent of crutch use on page 98 was closely similar to that on the earlier practice page, page 87. Practically all children in the EAR- and the DR-classes in these centers used the crutches on the two practice pages. That this was so is not strange since both pages were assigned during the time when the subjects were *required* to use their crutches. On this account it seems reasonable in the case of Center B to *estimate* crutch use on page 98, for which there are no data, as the same as on page 87, for which there are data. Some further justification for this practice is found in that fact that in Center B the records of crutch use for the DR-children are about the same on the two practice pages.

The actual data for crutch use in Center B are omitted, but the main points are represented in Figures 5 and 6, identical in construction with Figures 1 and 2 for Center A and with Figures 3 and 4 for Center C. Both sets of curves, based on frequency of crutch use (Fig. 5) and on per cents of pupils using crutches (Fig. 6), reveal essentially the same facts. During the Experimental Period proper 94 per cent or more of the examples were computed through use of the crutches, and 97 per cent or more of the pupils used the devices once or more. That is to say, such was the condition up to the end of this period when the End Test was administered. By that time both groups of subjects had begun to abandon the crutches. On that test nearly 20 per cent of the examples were computed without crutches in the case of the EAR-subjects, and nearly 30 per cent in the case of the DR-subjects (Fig. 5). Moreover, 10 per cent of the EAR-children and about 25 per cent of the DR-children took the test without once making use of their respective crutches.

Extraordinary losses in crutch use and in the proportion of chil-

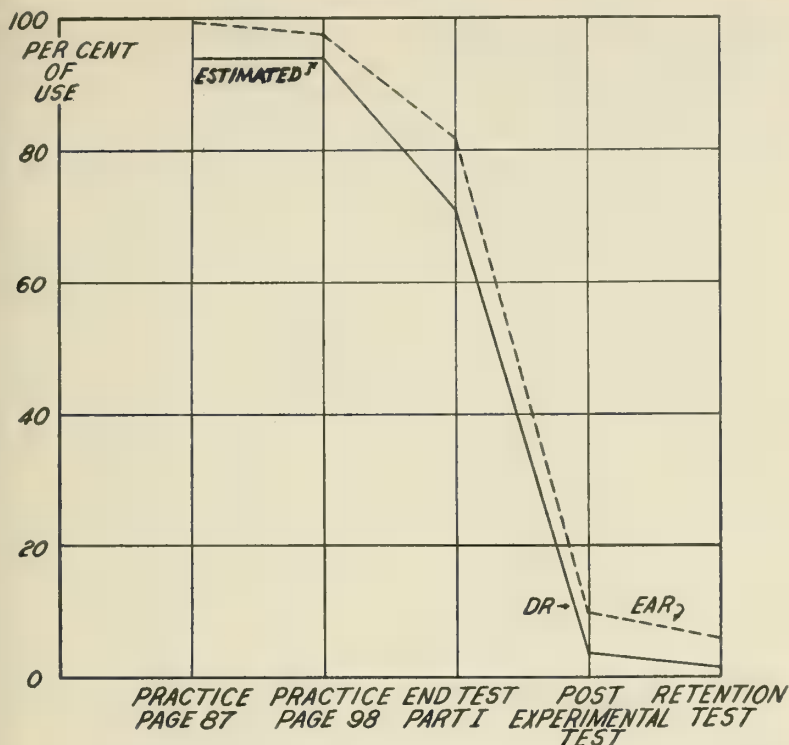


FIG. 5. Actual number of crutch uses divided by possible number of uses, at successive times of measurement.

dren using the crutches followed directions from the teachers to discard the devices (Postexperimental Period). By the time the Retention Test was given, only two examples in a hundred were computed with the crutch by the DR-subjects, and only six examples in a hundred by the EAR-subjects. Ninety-five per cent of the DR-children had surrendered their device entirely, and 86 per cent of the EAR-children had done likewise.

These findings for Center B are in essential agreement with those for Centers A and C, in showing that the crutches, once taught, did not necessarily continue in use. The differences among the centers are those of degree, Center B revealing a greater degree of independence than either of the other centers.

As to the usefulness of the crutches, again the evidence is to be inferred. The DR-crutch seems to have been more helpful than the EAR-crutch, as well as being easier to abandon.

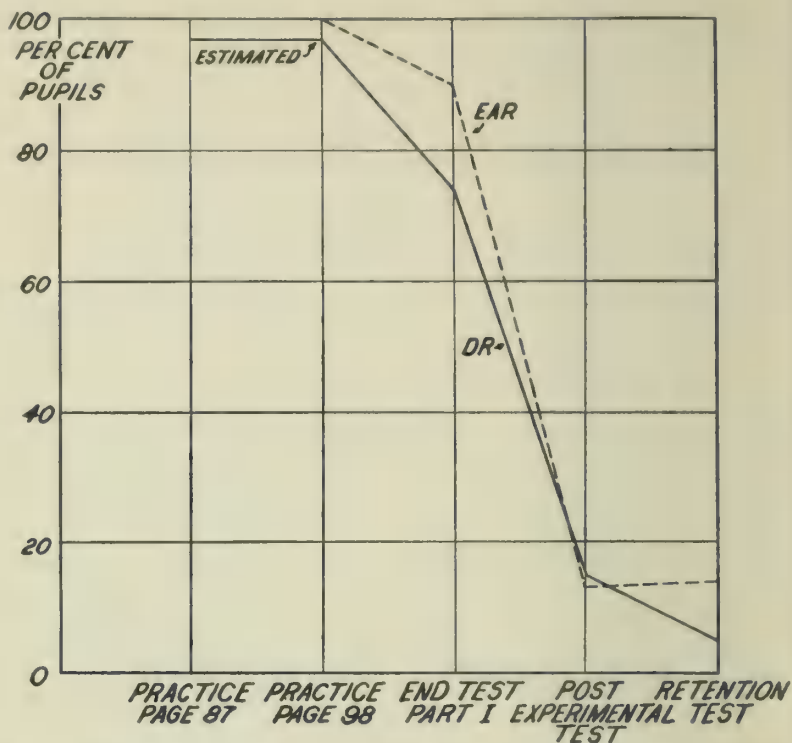


FIG. 6. Per cents of pupils using crutches, at successive times of measurement.

#### INTERPRETATION

It may as well be admitted that the writers were none too happy about the experiment in Center B and none too hopeful that the results would mean much. They fully realized the dangers in not being able to keep in close contact with the Center B teachers during the course of the experiment. Their fears respecting the effects of lack of supervision soon seemed to be warranted. First, as has been explained, one set of pupil work papers was not forthcoming. Then the second interviews had to be abandoned. Next, when the diaries were read, some of the M-teachers were found to have failed to teach borrowing, as directed, merely as a mechanical trick. And, finally, the skewed distributions of scores on measures collected in Interview I could not be accounted for. Taken together, these irregularities might have wrecked the whole study in Center B, for

differences relating to the experimental factors might have been reduced to nil.

But the anticipated disaster did not occur. On the contrary, when the data for Center B were treated systematically, the findings proved to be directly in line with those for Centers A and C. Apparently the experimental structure was sturdy enough to withstand the shocks of such discrepancies as there were between plan and execution. Or, stated otherwise, the differences between experimental factors—meaningful vs. mechanical instruction, decomposition vs. equal additions—these differences were real differences and were strong enough to reveal themselves in spite of the obstacles interposed by unintended interferences. The fact that the differences “came through” amidst the adverse conditions in Center B effectively forestalls any suspicion that the results secured in the other centers could have been produced by the novelty of the teaching devices employed. In a word, and to repeat what was said at the beginning of the chapter, the data for Center B add greatly to the reliability of the conclusions for the investigation as a whole.

1. The general findings in Center B agree with those in Centers A and C in showing:

- a. the outstanding success of teaching decomposition rationally,
- b. the greater difficulty of teaching equal additions rationally,
- c. the much greater transferability of skill in borrowing when taught by decomposition rationally,
- d. the insignificance of initial rate of computation as an indicator of real competence in borrowing, and
- e. the probable usefulness of the D-crutch for learning, the less certain usefulness of the EA-crutch, and the comparative ease of discarding both crutches in favor of shorter algorithms.

2. The findings for Center B agree with those for Center A in showing the greater effectiveness of EAM over DM in producing rate and accuracy of computation in the process taught. In this particular the findings support the results of previous research but disagree with those for Center C.

3. As for the educational significance of the differences in mean accuracy with C.R.'s of 3.00 or more, the following facts are pertinent. Converted to differences in per cents of accuracy, the twenty-five C.R.'s distribute as follows:

|                                 |             |
|---------------------------------|-------------|
| 4 per cent points or less ..... | 0 instances |
| 5- 9 points .....               | 7           |
| 10-14 points .....              | 12          |
| 15-19 points .....              | 5           |
| 20 points or more .....         | 1           |

On the standard of a difference of five correct subtractions per hundred, all twenty-five comparisons are educationally significant. On the more rigorous standard of 10 per cent, eighteen comparisons are educationally significant. The smallest per cent differences (range 5 to 13) were found for the End Test, both as a whole and on the borrowing examples only. The per cent differences on the Retention Test, of which ten are included in the tabulation above, ranged from 7 to 19, with all but one amounting to 11 or more. The per cent differences for the Transfer Test ranged from 7 to 20, with six of the eight differences equaling 10 or more.

## CHAPTER VI

### THE EXPERIMENTAL DIARIES

Considerable quantitative data have been presented to show the effects of four different plans of instruction upon the learning performance of children beginning a study of borrowing in subtraction. Figures alone cannot supply all the information needed to compare the effectiveness of the procedures employed. Also of considerable importance are the classroom experiences of the teachers operating under the varying conditions of instruction. Important questions such as the following need to be raised: Is there any evidence to show that rationalization by decomposition is more, or less, difficult to teach than is rationalization by equal additions? Which of the four procedures are to be considered satisfactory from the point of view of the total effect upon the children? Were the crutches helpful; and, if so, which children benefited most from them? For the purpose of answering questions such as these the teachers participating in the experiment were asked to keep experimental diaries. In order to get unbiased reactions no outlines were given the teachers; they were asked merely to record the significant happenings for each of the experimental periods as they saw them.

The data from these diaries will be presented by experimental centers. The contents of the diaries cannot be presented in full. Instead, specific entries will be selected as they contribute to questions of experimental interest.

#### EXPERIMENTAL CENTER A

*I. What is the effect of the use of mechanical procedures upon children with two years of meaningful instruction in arithmetic?*

As explained earlier, Center A was the only one which provided a real opportunity to study this question. The diaries of five of the six teachers of M-classes contained anecdotes indicating the children's dissatisfaction with the explanations offered. The forms in which this resistance was made manifest are indicated by the following excerpts from the diary records:

#### *DM-Section—Teacher A*

Lesson I. "The class showed interest in the new type of work but the children were puzzled at going into it with no type of explanation.

After the first example was demonstrated, comments such as 'I don't get it,' and 'I just don't see it' were heard from all sides."

Many specific questions were asked by the children in this class for the purpose of eliciting a rationalization for the procedure. The entry for the second day illustrates this point:

"The pupils were concerned about whether you decreased the tens by one or how many. They asked, 'How many smaller do you make the tens?' 'How do you know how much larger to make the ones?' 'If you make the tens' line smaller, why don't you make the hundreds' line smaller too?' There was more confusion."

Resistance gradually subsided by the third day. The diary record for that day notes: "The class seemed to be settling down as a whole to this kind of subtraction and some are mastering it.—Only two wanted to ask for help."

#### *DM-Section—Teacher B*

Lesson I. "The children are enthusiastic about the new work. When individual work began, cries of 'I don't know how to do this' were heard. Individual explanations [verbal formulae] were given with still a look of bewilderment. The teacher felt that they would have understood if only they knew where the ten came from. About half the class did fairly well; the rest had no conception of what to do."

This resistance was short-lived, however, because the entry for the second day notes: "No comments made [by class]." The work evidently was less attractive than usual for some of the children because the record for the sixth day contains the statement, "Several who did fairly well in arithmetic in the past fail to make numbers smaller after borrowing."

#### *DM-Section—Teacher C*

Lesson I. "About half the class had trouble. All wanted to know where the numbers used to make larger numbers came from."

On the second day the teacher intercepted the following note from one pupil to a classmate: "Mrs. ——— acts funny. She won't let us play any games like we used to." The teacher concluded that they were evidently tired of the verbal formula.

Some of the children attempted to provide their own meaningful experiences. First day: "One child asked for concrete materials; two others made marks in their tablets to see whether they had made numbers smaller correctly." As with the other DM-classes, the out-

ward manifestations of resistance disappeared by the third day. At this time the teacher wrote that difficulties had been reduced for all but five children.

*EAM-Section—Teacher A*

In this class the subtraction formula supplied by the teacher failed to satisfy the children, and they tried to answer their own questions. The results were not adequate, however, as indicated by the diary note for the first day: "As explanations developed the children became more and more confused."

By the third experimental period these children, too, had accepted the pattern presented by the teacher. The entry for this lesson reports: "Teacher explained two examples; no questions. Children asked permission to explain others."

*EAM-Section—Teacher C*

The children in this class immediately attacked the part of the teacher's "explanations" which did not explain. Many questions were asked. Excerpts from the diary indicate the reactions:

After the first explanation the question asked by most of the children was, "Where did you get the 1 to make the 5 a 15 and the 1 to add to the 2?" Some of the children do not understand why you need a 1 for the 5 (to make 15) and a 1 to make the 2 a 3.

Dissatisfaction with the teacher's explanations is also revealed by the fact that some children asked their parents these questions and had to be cautioned by the teacher to confine their homework activities to practice in the basic combinations. No mention is made in the diary of resistance to the verbal formula used by the teacher after the second day.

*II. To what extent can children who have had two years of meaningful instruction in arithmetic discover for themselves the rationalization for borrowing?*

In the preceding paragraphs evidence was presented to show that many children in the M-sections actively sought rationalizations for the procedures employed. Evidence also was introduced to show that in several instances children tried to find for themselves answers to the questions which the verbal pattern supplied by the teacher did not provide. Here interest lies in the degree to which the children could apply their earlier meaningful experience with hundreds, tens, and ones to an explanation of borrowing in subtraction.

tion. Diary records for three of the six classes in the M-sections showed that some progress was made.

#### *EAM-Section—Teacher B*

The verbal pattern given to the M-sections did not refer to ones, tens, and hundreds; but this class had regularly used these terms in the analysis of pre-experimental number tasks. From the very first lesson in the subtraction study they employed the familiar terms to supplement the verbal pattern supplied by the teacher. The following entries show the extent to which the principles of place value were used in arriving at a more meaningful explanation:

Lesson I. Variations in language pattern used by the children: "I subtract my ones' column first."—"I add one ten to 5."

"One child invented an explanation. 'Borrow one ten from the tens' column'; she hesitated for minutes; then she asked, 'Why do you change the 2 [in subtrahend] to a 3?' She showed that she was much confused."

The phrase "borrow from the tens' column" was taken up for a day or two by a number of children, but no one was able to explain the increase in the tens' column in the subtrahend.

Not all the children in this class had the same background of meaning. On the second day the teacher reported: "One child could not understand why the children used the term *tens* when it is not in the number pattern." Upon still another occasion one pupil was explaining an example to another who was having difficulty. His explanation developed as follows: "Betty, add one to this one [pointing to the tens' column]; 1 plus 1 equals 2." Another pupil interrupted at this point, "We need a better explanation. He means to add one ten."

In neither of the other two EAM-classes is there any record that the children altered the words of the verbal pattern or used the terms *ones*, *tens*, and *hundreds* in making explanations.

#### *DM-Section—Teacher A*

Lesson I. "After the second example one child wanted to know whether you always made the tens' number smaller. When the explanation [formula] was given, he wanted to know *why*. He was answered by use of the formula and did not press for another explanation but did not seem satisfied.—Betty wanted to know where you got the one. When asked, 'Which one?' she said that it was the one that you made the number bigger with. Billy said, 'Maybe Betty doesn't understand that you took the one from the tens' line.' I was anxious to know why or how

he knew that and so I said, 'Why, what makes you think so, Billy? I haven't said anything like that and neither has the book.' He replied, 'I just figured that out for myself.' "

In the absence of encouragement on the part of the teacher this good beginning was not followed up. On the next day this lead was abandoned, and the children were pursuing other and more remote clues. The record:

"The pupils were all concerned about whether you decrease the tens by one or how many. They asked, 'How many smaller do you make the tens? How do you know how much larger to make the ones? If you make the tens smaller, why don't you make the hundreds smaller too?'"

*DM-Section—Teacher C*

Apparently not much progress was made by the children in this class with explanations of borrowing in terms of procedures with tens and ones. The only note in this connection appeared on the fourth day. "One child figured out part of a rationalization for himself. He figured out from what the class was doing that we added a ten each time we couldn't subtract. However, he was unable to explain the changes in the tens' column."

In summary, then, it may be said that none of the children in any of the sections was able to arrive at a complete rationalization of the borrowing technique. On the other hand, the nature of the manipulations in the ones' column seemed fairly clear to a considerable number of the pupils. Several even reasoned that the ten thus used must come from the tens' column, but the idea of actual transposition of a part of the tens' minuend to ones (in D) was never clearly formulated. How much further these children might have gone with encouragement and guidance on the part of their teachers is a matter for speculation.

*III. What evidence, if any, was there in the M-sections of the need for an intermediate level of performance?*

None of the EAM-teachers reported that any of their pupils invented "crutches" to help them with the subtraction, but all the DM-teachers reported such incidents. After the third lesson Teacher A wrote: "One child wanted to know why we didn't rub out the number when we made it smaller and put the other one in; then they wouldn't forget so much." This same teacher also told of one child who actually erased the tens' figure and wrote in the new digit. She reported that this crutch was abandoned in the next lesson.

The crutches used by the pupils of Teacher C showed more variety. For the first day she submitted the following report: "One child rewrote the example  $\begin{array}{r} 65 \\ -28 \end{array}$  as  $\begin{array}{r} 5 \ 15 \\ -2 \ 8 \end{array}$ . He said that he could do it better that way." On the second day the following entry appeared: "One child wanted to erase the number in the tens' row and write the smaller number. Said he couldn't think of it unless he saw it." Again, on the third day: "Six children are using fingers and drawing strings of beads in the back of their tablets in order to add and take from the combinations."

The invention of crutches certainly may be taken as a positive indication of need. The absence of such devices, however, cannot be taken as a sign of *no* need. Invention is chiefly a function of insight and personal initiative. Hence, if this statement be true, children standing in the greatest need of crutches are not likely to demonstrate a considerable amount of either quality.

One of the teachers whose pupils worked out no crutch devices included the following comment in her last diary record: "The children who made the low scores are the individuals who need 'crutches' and 'clinchers.' These are not given in the EAM-instructions; consequently, such children are left to fail."

*IV. What indications were noted by the teachers of R-sections that the "crutch stage" was a helpful but an impermanent phase in learning to borrow?*

All three of the DR-teachers made rather frequent reference to the value or to the persistence of the crutch, but only two short comments were found in one of the diaries of the two EAR-teachers. This evidence is hardly sufficient to validate the conclusion that rationalization by means of the crutch was found to be more useful to the D-classes than to the EA-classes; but it warrants the further investigation of this hypothesis in other centers.

#### *DR-Section—Teacher A*

This teacher's comments make it clear that the children in her class experienced very little difficulty learning the new subtraction process. It was the only class in which a number of children were ready to abandon the crutch as early as the fourth lesson.

Lesson IV. (Comments of children) "Do we still have to write the number picture?"

Lesson VI. "Six children left off the 'number pictures.'"

Lesson VIII. "Some wanted to do without a crutch. They think it takes too long."

Lesson IX. "All used the crutch except two. These know how to borrow thoroughly. Four had to use Step 2 (extended crutch)."

Lesson XV. "All can use the third step (no crutch) and like it. Many wanted to know when they could stop using the crutch."

After End Test. "Children did examples without the crutch. A few (three) were lost. Children's comment: 'We can go much faster.'"

One month later. "Just two use the crutch. They do not wish to use it, but they do not make as many mistakes if they do."

Two months later. "I am very pleased. The crutch is never mentioned unless I mention it."

#### *DR-Section—Teacher B*

The teacher of this class felt that the work moved slowly. Part of the difficulty was attributed to poor attendance. At the time of Lesson VII she estimated that twenty-two children still did not understand the process.

Lesson VIII. "Examples were worked exactly as on the first and second days." This note indicates that a rather elaborate form of the crutch was necessary.

Lesson X. "Crutch still necessary."

Lesson XIII. "Crutches! Most of the group seem lost without them."

Lesson XV. "All use the crutch in borrowing."

There was no further diary account of the use of the crutch. Whatever the cause, it is clear that the learning pace for this DR-class was much slower than for the other classes in the section. The testimony of the teacher indicates that the crutches were needed to the very end of the instruction period.

#### *DR-Section—Teacher C*

Two learning problems complicated the task of Teacher C. The same epidemic which caused so many absences in Class B also retarded the progress of Class C. In addition to poor attendance this teacher had twelve children who were new to the school and did not have the background of meaning possessed by the others. The teacher found it "difficult to explain the *why*, *what*, and *when*." The number language used in the classroom had no meaning for the new children.

In the second lesson this teacher discovered that the first children to omit the crutches were the ones who did not understand the procedure. This observation was verified in later lessons. After Les-

son V she again wrote, "As a rule the weak pupils do not use the crutch. One such pupil missed seventeen out of twenty examples."

Other observations concerning the use of the crutch in Class C:

Lesson VII. "Only two children failed to use the crutch. I am sure that they know how to borrow, but they are simply careless."

Lesson VIII. "None of the children used Step 2."

Lessons IX and X. "I am delighted to see so many of my children trying to figure things out for themselves. . . . Children termed 'dependent' at the beginning of the experiment are now making fewer mistakes than the children who never asked questions."

Lesson XI. "All of the children used the crutch. The results are encouraging."

Lesson XII. "I am pleased with the way the children remembered to use the crutch after the Thanksgiving holidays."

Lesson XIV. "Most of the children are using only Step 3."

After the End Test. "Many children said that they were going to leave off the crutch because they could do the problems with their eyes closed."

Postexperimental Period. "I am rather delighted with my results. Nobody ever uses the crutch. They simply asked whether they couldn't stop and do it the short way."

#### *EAR-Section—Teacher A*

As mentioned previously, not much evidence concerning the need for the crutch was provided in this report. One entry, however, was very significant:

Lesson VI. "The children working correctly used the crutch every time. The ones making errors did not use the crutch."

Here we find support for the evidence supplied by DR-Teacher C.

Toward the end of the instructional period EAR-Teacher A noted that the children were using the crutch less than formerly. This comment, too, agrees with evidence reported earlier showing that crutch forms are abandoned with the development of skill at the higher levels.

*Summary:* The evidence from both the M- and the R-sections strongly supports the point of view that the borrowing crutches are valuable teaching aids. In general, it has been shown that the slower the children the more the crutches are needed as intermediate algorithmic forms. The anecdotal records also give further proof of the ease with which the crutches are abandoned when the learner has established control over the verbal pattern.

*V. To what extent did the various procedures employed measure up to the teachers' demands for an effective method for teaching borrowing?*

A word of explanation may give point to the purpose of this question. The strategic place of each teacher in the classroom provides the best opportunity to notice the impact of the learning demands upon the pupil and to observe his adjustments to those demands. This intimate relationship also makes her the best judge of the quality of his behavioral reorganizations. Indeed, in the present state of educational research she is almost the sole source of evidence on the total effect of the learning upon her pupils. If the learning demands exceed the capacity of the children, she is quick to note the evidences, in terms both of extra teaching effort on her part and of the increased tensions and evasions on the part of the learners. She is sensitive to keen enjoyment stemming from successful accomplishment, and notes the nonfunctional character of pseudo learning. For all these reasons the writers feel that this experimental report would be incomplete without the recorded judgments of the teachers themselves.

#### *DR-Section—Teacher A*

At close of Experimental Period. "All can use the third step and like it. Many wanted to know when they could stop using the crutch. Experiment beneficial to class and teacher. Boys and girls liked the work. Have learned to borrow and understand the meaning."

At close of Postexperimental Period. "This method for borrowing is the best I have ever seen. It is easy and meaningful. I have had better results with this class than I have ever had before. Children really like to work subtraction."

#### *DR-Section—Teacher B*

At close of Experimental Period. "The poor attendance before and during the experiment handicapped the group and the teacher. However, all the children, even E—— N——, are interested in arithmetic and enjoyed borrowing in subtraction."

#### *DR-Section—Teacher C*

At close of Experimental Period. "This experiment has been illuminating. I must alibi and say that it has been difficult to teach anything with seven and eight children absent every day."

After End Test. "I am beginning to hear remarks like this, 'I used to think this stuff was hard.'"

At close of Postexperimental Period. "I am rather delighted with

my results. Nobody ever uses the crutch. . . . The mistakes made in subtraction problems are due to poor comprehension in reading."

*DM-Section—Teacher A*

At time of last entry (20th lesson). "During the Postexperimental Period we have had some subtraction examples at least three times a week, usually in a mixed practice test. I have continued to give individual help to the ten pupils who have constantly made errors. But even now there are quite a few who make mistakes unless they have help." (Note that this "instructional help" was contrary to directions.)

*DM-Section—Teacher B*

At close of Postexperimental Period. "During this period we worked on the third and second multiplication tables. Also we reviewed our carrying and subtraction as we had taught it in the Experimental Period. Having been working exclusively on subtraction for such a long period of time, some of the children were confused as to when to add and when to subtract. Some looked at the subtraction examples similar to those we had been working on and seemed not to recognize the type examples. The teacher repeated the formula used and asked them if they didn't remember these examples. They did. During this period the formula was repeated for those who hadn't gotten it in the Experimental Period, and it is hoped some learned it."<sup>1</sup> (Note here again that the teacher ignored directions.)

*DM-Section—Teacher C*

No summary statement given.

*EAR-Section—Teacher A*

Concluding comment: "With much repetition and close supervision the children have made a good start with EAR." (Note: *a start!*)

*EAR-Section—Teacher B*

Summarizing comment. "Persistent repetition and careful supervision needed to obtain a fair degree of accuracy."

<sup>1</sup> Two other M-teachers in their day-by-day comments testified to the ease with which the verbal formula became dissociated from the solution of borrowing examples:

EAM-Section; Teacher B. (Note on last lesson.) "Many low scores are made by children who give perfect explanations. . . . This condition of giving perfect explanations and yet failing to hold constant scores (in written work) has been interesting throughout the experiment." Evidently the "explanations" were merely verbalizations of unintelligible word patterns.

DM-Section; Teacher C. In Lesson VII this teacher reported that several children regressed somewhat in quality of work. She gave the special help of the type permitted, then reported, "Neither asked for further explanations nor intimated that they didn't know what to do when I said the verbal pattern, but when left alone they just didn't 'carry on.'"

*EAM-Section—Teacher A*

Conclusions. "As a matter of interest I checked the final test. It presents an entirely different picture of the group—thanks to much and thorough drill—and a much more hopeful one. . . . The results of the children's test show how much they have benefited from the experiment. . . . I could not teach arithmetic without putting meaning into dull figures."

*EAM-Section—Teacher B*

Summary statement. "Such students as . . . [names six] have ability to reason and think problems through. I feel that they have been in a fog during the experiment; their keen memory was their guide. The children who made the low scores are the individuals who need 'crutches' and 'clinchers.' These are not given in the EAM-instructions; consequently such children are left to fail.

"During the Pre-experimental Period the entire class was making progress. More accuracy and understanding were shown by their using the terms *ones*, *tens*, and *hundreds* in the analysis."

*EAM-Section—Teacher C*

Note at close of Experimental Period. "My students have thoroughly enjoyed working in this experiment though they didn't realize it was an experiment."

*Summary:* In view of the varying quality and quantity of the comments submitted it is necessary to exercise care in making comparisons. It may be said, however, that two of the DR-teachers were enthusiastic about the method used. The third comment may be classed as noncommittal. On the other hand, both DM-teachers making final statements were highly critical of its effectiveness. This judgment is well corroborated by the evidence presented in the quantitative data (Chapter III).

The two EAR-reports are too brief to be assessed properly, but they do create the impression that whatever progress was made was obtained by virtue of extended teaching effort. One of the EAM-reports is sharply critical of the procedure; one is ambiguous (praises results but condemns the nonmeaningful character of the process), and one is noncommittal.

In general the results seem to favor DR, but judgment must be suspended until the diaries from the other Centers are reviewed.

## EXPERIMENTAL CENTER C

The diaries of the teachers in Center C were of experimental importance because they served as controls for the results obtained

in Center A. They also provided important information on the extent to which rationalization may be employed fruitfully with children having a minimum of previous instruction in meaningful arithmetic. A total of fifteen diaries was received from the teachers in this center: five from EAR-classes, four each from EAM- and DR-classes, and two from DM-classes.

As explained earlier, the arithmetical background for the pupils in Center C was quite different from that in Center A. A number of general comments found in some of the teachers' diaries reveals something of the limited amounts of arithmetic taught in Grades 1 and 2 and the rather mechanical character of this early instruction. Of the six teachers reporting on M-procedures only one EAM-teacher suggested that the proposed program for teaching borrowing mechanically differed materially from the usual kind of teaching. Several references indicated the limited amounts of arithmetic instruction in Grades 1 and 2. One teacher remarked that if borrowing is to be taught as early as November in the third grade, more arithmetic will have to be taught in the lower grades. She continues with the personal observation that she is opposed to much arithmetic in Grade 2. Another comments that the children in her room had not had the basic combinations with sums and minuends beyond 10; hence she had to institute special drills. Several teachers referred to feelings of "being rushed" at having to begin borrowing so early.

*1. What evidences of resistance to mechanical procedures were shown by the children in Center C?*

The diaries from Center A indicated that the children who had become accustomed to full explanations for the operations in arithmetic offered considerable resistance to attempts to teach them by mechanical procedures. In many instances the pupils so taught attempted to work out for themselves the explanations which the teachers withheld. However, the behavior thus described can be attributed to a particular kind of training only if it can be shown that children under similar circumstances but with different backgrounds react differently. Center C provided such a control.

*DM-Section—Teacher A*

No comment.

*DM-Section—Teacher B*

Pre-experimental note. "Most of the children never question anything they are told, nor seek to understand any problem that is not clear to them, nor do they follow directions well."

After the first lesson. "No questions were asked as I worked the examples on the board and told the class 'how to think.' The children tried hard to remember how the preceding problems were worked."

*EAM-Section—Teacher A*

This is the one teacher of an M-class who spoke of having regularly taught arithmetic with meaning. On the first day she wrote: "Not nearly so many complaints as I had expected." However, on the fourth day the following entry appeared:

"As a whole the children want to learn *how*. Those who were fairly good in arithmetic that we had prior to the experiment are falling far below in this work. Some ask for a more thorough explanation and after I repeat the formula and have them follow with it, they go back to their work in the same blank state of mind. What are they thinking of me?"

*EAM-Section—Teacher B*

First day. "No question as to the method arose. It was merely accepted and applied. Very few questions were asked—not more than six." The children's questions reported had nothing to do with the rationalization of the procedure.

*EAM-Section—Teacher C*

First day. "Children asked no questions about why in the example  $64 - 36$  we think 6 from 14 leaves 8."

*EAM-Section—Teacher D*

The diary of this teacher contains several verbatim reactions of the children. First day comment of a pupil: "Gee, that's easy now. Just say, 'Think the number,' and keep on going." Later in the same period there is a record of one child's response to a question of a classmate: "Goodness, don't you know [how to do the exercise]? Look, I'll tell you what to do. Take the bottom number from the top number. If the number isn't as big as the bottom number, think it to a larger number. I'll show you."

This last entry shows how completely the children accepted the "explanations" which did not explain. Taken as a whole, the records just summarized make it clear that the critical thinking shown by many children in Center A came as the result of the particular kind of training they had received prior to the experiment. Children used to "thinking patterns" seldom ask for anything more.

*II. To what extent did the children discover for themselves the mathematical principles underlying borrowing?*

Inasmuch as the children of the M-sections were satisfied with the verbal formula supplied by the teacher, it may be anticipated that they would make little effort to rationalize the process in terms of mathematical principles. Such was indeed the case. Only one mention was made of any attempt in this direction during the instructional period. After the second lesson DM-Teacher B recorded the following anecdote:

One little boy who is not a repeater spoke of borrowing one from the tens. I immediately "corrected" him. He was evidently doing some rationalizing on his own. He is perfectly capable of it, and I don't think he had much, if any, outside help.

The difference between the children in Centers A and C in the matter of independent rationalization is further emphasized by a note written by this same teacher during the Postexperimental Period. "I have noticed that several children seem to be getting the idea where that '1' comes from, but they are the brighter and more advanced pupils." This is to say that a few of the brighter children were discovering, three months after the beginning of the experiment, what many of the children in Center A worked out for themselves in the first few lessons.

*III. What evidence was there in the M-classes of the need for an intermediate level of learning?*

It is hardly to be expected that the evidence supporting the need for a particular kind of learning aid would be as pointed in Center C as in Center A. In the first place, the children had had little experience with crutches and lower levels of performance. Under such circumstances pupil inventions of variations in the basic algorithm would be exceedingly unlikely.

In considerable measure the same conditions influenced the teachers' remarks. A teacher who has taught exclusively by mechanical procedures is unlikely to set up a specific variation in the learning pattern she had continuously used. From these diaries it cannot be told how many times the expressions "careless errors" and "too slow to catch on" concealed the real needs of the children. Only diary entries having specific implications with respect to the problem are reported below.

*DM-Section—Teacher A*

Lesson I. "They had trouble remembering to think the left-hand number as anything different at all."

Lesson II. "Some of them still have trouble changing the tens' number."

Lesson IV. "They are still forgetting to think the second number as anything different."

*DM-Section—Teacher B*

Lesson I. Many and varied errors. In struggling with the new difficulties some children lost control over simpler aspects of subtraction learned earlier, e.g., the child who was found to be saying, "Eight from 4 leaves 4." Teacher's comment: "It's hard to see them struggle with work which is so bewildering but which could be understood so easily." Difficulties were not confined to the slow pupils. "Five of my brightest children had trouble, while some of the more docile 'average' children 'caught on' immediately."

Lesson II. "Two children simply could not remember to think of the figure in the tens' place as 1 less, but others also thought of the figure in the hundreds' place as 1 less."

Lesson III. "There are still five children (plus those counted as 'hopeless') who forget to think of the tens' figure as 1 less or who also think of the hundreds' figure as 1 less. They are not consistent in their mistakes. At other times they think a 1 in front of the tens' figure as well as in the ones' column."

Lesson IV. "The same mistakes are made over and over. There seems to be no way to 'explain away' the difficulties."

Lesson V. "Ten children (plus the 'hopeless' cases) still forget that they have borrowed a ten in the previous step."

Lesson VII. After Thanksgiving holidays. "About one fourth of the children seem to have forgotten how to borrow. The most common mistake was forgetting that a ten had been borrowed in the first step."

Lesson VIII. "Ten children still seem unable to remember the formula from day to day."

Lesson IX. "I noticed in today's work that twelve of the children, instead of borrowing in the first step of a problem, make the mistake of taking the upper number from the lower. Evidently the thought pattern is still not well established in spite of all the drill I have attempted to provide."

Lesson X. "There are still about ten children (plus the 'hopeless') who simply seem unable to master the thought pattern for working subtraction problems with borrowing. I can see no evidence of progress in the work of any of these children. The class as a group has little difficulty working problems such as those used in the lesson today."

Lesson XI. "Same old mistakes are made over and over! I have tried to work individually with those who need my help most, but the children seem to be in an 'impenetrable fog.'"

Lesson XII. "I have noticed that a good many children can work the problems correctly if I stand right over them to insist that they work carefully, but when left to work absolutely 'on their own,' they forget that they have borrowed a ten, or they make other typical mistakes."

Postexperimental comment. After noting that some of the children seemed to be getting some idea of where the borrowed 1 came from, Teacher B wrote, "Most of the children, however, seemed to have no better understanding of the process."

#### *EAM-Section—Teacher A*

Lesson II. "Children who are good in arithmetic having a grand time. Majority of class doing work *incorrectly*."

Lesson III. "Too many still are lost as far as knowing what it is all about. Extremely slow pupils never get through, and when they do, everything is all wrong. Majority of class does not work correctly. Little mistakes continue; big mistakes continue."

Lesson IV. "Something is wrong. I believe things should certainly show improvement by now."

Lesson V. "Children becoming more familiar with formula and are working hard to master these examples. Children like their work but miss things they have to leave off for extra arithmetic." (Note that the teacher was apparently giving extra time to arithmetic.)

Lesson VI. "No difficulties among better pupils. Middle and lower groups are still groping in the dark. Child's comment: 'I forget to think of so-and-so as so-and so.'"

Lesson VIII. "Same mistakes made daily as I listed on second day. These are easily corrected by some pupils and, as yet, never have been understood by others."

Lesson X. "I am convinced that pupils are being pushed. The smarter ones can take it, which is only normal."

Lesson XIII. (After several days of encouraging reports). "Progress about the same; just as I begin to feel encouraged I find that I have no reason to be. Very discouraging to teach and have so many up-and-down results."

Lesson XV. The entry for this day comments upon the unsystematic character of the errors. "It [the mistake] may follow an example which they have worked correctly. This is often the case."

#### *EAM-Section—Teacher B*

Lesson I. Difficulties limited to a few children.

Lesson III. "More mistakes were made and the examples seemed harder to the pupils. The zero was troublesome."

(The teacher continued to report no special difficulty encountered, but on a number of occasions fourteen of the thirty-six children had correct answers for fewer than half of the examples in the daily exercises.)

*EAM-Section—Teacher C*

Lesson I. "Several children were asked to give explanations and seemed to understand the steps. All insist on explaining the long verbal pattern, however." Shorter verbal pattern used with three examples.

Lesson II. "Children used shorter method. Many mistakes made by all the group on extra practice set number 2. Forgot to add a ten to the tens' column in subtracting."

Lesson III. Longer pattern used by four children; others encouraged to use shorter formula. Chief error: failure to increase subtrahend in tens' place.

Lesson IV. "Asked children to use shorter verbal pattern or direct method of thinking."

Lesson VIII. "Shorter thought pattern being used by greater number."

*EAM-Section—Teacher D*

Lesson I. Comments of pupils: "This sure is hard. I can't think that way." "I see it better when I work at the board. May I work another at the board?"

Lesson IV. "This work was done very quickly by the good working group. Only one example wrong in fourteen papers. The slower pupils missed practically every example. Just poor workers; grade too hard for them."

Later comments in the report indicate that good work was done by twenty pupils. The remainder (about ten children) were described as "hopeless." As in all EAM-reports, the chief error was caused by failure to increase in the tens' column after increasing in the ones' column.

*Summary:* The teachers' failure to cite more specific evidences of the need for ameliorating the learning difficulties of the children was anticipated. It is significant, however, that in four of the six DM-diaries figures were given to show that approximately one third of the class was experiencing extreme difficulty. This seems to be a very high ratio for borderline performances and below.

*IV. What indications were noted by the teachers of R-sections that the "crutch stage" was a helpful but impermanent phase in learning to borrow?*

*DR-Section—Teacher A*

Lesson II. "Introduced crutch and long verbal pattern."

Lesson III. "Very little difficulty with zero."

Lesson IV. "Children learning well the process of borrowing."

Lesson VII. Comment from child: "Mrs. ———, do you want me to write these little 1's? I can do just as well without them."

Lesson VIII. "Crutch and Step 3 used by all children."

Four days after Interview I: "We work subtraction examples the grown-up way. Children experienced no difficulty. Occasionally a child would get the wrong answer because he forgot that he had borrowed. Examples were corrected with the crutch."

*DR-Section—Teacher B*

Lesson I. "All seem to understand way of showing 'number picture' and reason for so doing."

Lesson II. "Continued with Step 1; developed verbal pattern. Few mistakes. . . . Children usually able to correct errors."

Lesson III. "Omitted Step 1 as children seemed to understand."

Lesson IV. "Continued to use Step 2."

Lesson V. "Children quite ready to begin with shorter form. Some already using it. Felt that children did very well."

Lesson VI. "Some failed to show 'number picture' until asked about it."

Lesson VII. "Mistakes because about ten pupils forgot to show their 'number picture.'"

Lesson VIII. "Very good work done on this page. A few children still using Step 2. Mistakes made were caused by not showing the 'number pictures.' As soon as shown, mistakes were corrected. These pupils absent during first days."

Lesson IX. "Practically everyone now using Step 3."

Lesson XII. "Did well. All but two have learned to borrow."

Subsequent lessons: "Very few mistakes. All used the 'number picture.'"

Postexperimental Period. "The children thoroughly enjoyed learning to subtract a new way without using the 'number picture.' Occasionally several children would forget and show the picture, but as a whole the transition was made quite easily and rapidly."

*DR-Section—Teacher C*

Lesson II. "Review of Step 1; response fairly quick and general. Step 2 used for some of fastest."

Lesson III. "Step 2 used. Five children very confused. Rest made few mistakes."

Lesson VI. "Much improvement shown. No use of sticks or finger counting."

Lesson VII. "No trouble going from Step 2 to Step 3. Twenty-one (of forty-one) pupils had perfect papers. Only three had serious mistakes."

Lesson XII. "All but two working by Step 3."

Comments in following lessons do not mention use of crutch.

*DR-Section—Teacher D*

Lesson II. "Used Step 1 again. Paper check revealed that nine did not understand. [Of these] three don't know how to subtract in tens' place; two borrowed correctly but added in tens' column."

Lesson III. "Step 2. All but four seemed to know how to make the 'number picture.'"

Lesson XV. "About half the children do not need 'number pictures.'"

The smooth progress of the course of learning to progressively higher levels of performance pictured by the DR-teachers stands in sharp contrast to the difficulties reported by most of the M-teachers in the early stages of the learning. It seems clear that the shock of the new learning is sharply reduced by the use of intermediate levels of performance. The evidence for this conclusion, first presented for Center A, is further substantiated by DR-Teacher B as she revealed the increased number of errors resulting from the failure of some children to use the crutch during the early part of the learning.

Several of the teachers of EAR-sections gave excellent descriptions of children's efforts to learn to borrow by a rationalized procedure. These reports were especially valuable because corresponding data from Center A were too fragmentary to be of much worth.

*EAR-Section—Teacher A*

Lesson I. "I set up the 'number picture' on the board for the example,  $65 - 28$ . Ninety per cent of the class followed with understanding."

Lesson II. "I began the lesson as I had the first day. Many were ready to explain perfectly. The weaker students began to feel a bit more confidence; but when they tried explaining several examples, difficulties arose.

"Adding a ten to the ones has always been a mental struggle for at least twenty of my children. I took the group that didn't understand into a different part of the room. We took actual objects and practiced adding them until adding in the ones' column became automatic. Then their difficulty was adding a *ten* in the tens' column. They would want to say, '10 plus 2 tens is 12 tens.' With all my effort I haven't been able, through the entire period, to get the figure in the tens' column correct."

Lesson III. "Review beginning with Step 2. (Used Step 3 in latter part of lesson.) Several examples were put on board for the children to give verbal pattern and 'number picture.'—[For children not understanding the principle] I would give the verbal pattern and demonstrate with objects and they seemed to grasp it [the principle]. I would let them give the verbal pattern and put in the crutch, and almost every time the tens were wrong. I find that I have seven children who cannot *see* the

right answer. They feel that it is something very different from simple addition and simple subtraction. With the addition of 10 ones and a ten they feel that they have entered into a mystic realm where their previous knowledge of addition and subtraction won't function. There is so much confusion in their minds about adding that they add in the hundreds' column, too.

$$\begin{array}{r} 943 \\ 217 \\ \hline 1126 \end{array}$$

"After making every sort of mistake in the 'number picture' one little girl said, 'This won't hang onto my brain.'

"This mistake was made in five of the seven papers of children who haven't seen the light: 'Now I can't take 15 tens from 8 tens and so I don't need to add a ten to 5 tens.'

$$\begin{array}{r} 481 \\ 154 \\ \hline 7 \end{array}$$

"Another child after correctly increasing the 5 in the subtrahend to 6 said, 'That will be 60, and I cannot take 60 from 8 . . . and so the example must be wrong.'"

Lesson IV. "Page 86 seems to be less confusing. One child said 'I've wanted to know where you get that ten. Whose is it? Can you take a ten any time you want it and add it any time?'

"Five or six of my weakest children are doing better than at any time since they have been in the third grade; three average students find it [subtraction] most difficult. Not a single child that I'd rank above average or good has had trouble."

Lesson V. "Thought children did rather well. [Work] not difficult but a strain for those not any too strong in arithmetic.

Lesson VIII. "Review after holiday. Process recalled easily by most of group. Those not good before are not good now. About five children have to go back to second step and are not able to say the process in the third."

Lesson IX. "No tendency to leave out the crutch. As long as all examples had to have 10 added in the ones' column the process was beginning to be mechanical and the children felt secure. As soon as [examples without borrowing] appeared, there was a feeling among the weaker students that they weren't sure of the process."

Lesson XI. "Best work as a group was done today. However, a few children will never grasp the facts, it seems. I found with one child the confusion came in the fact that you add in the ones' column, then you subtract. I let her do the addition in both ones' and tens' columns and insert the crutch marks. Then she did the subtraction throughout the example and she seemed to understand it. Her confusion was in mixing the operations. This may not be a good method, but it worked."

Remaining lessons: Only three or four children faltered in the procedure. These were behind in other work too. It was found that some children could correct errors they had made by looking over their papers.

Postexperimental Period: "Some of the children seemed to get a much clearer insight into the procedure when we were 'coasting' along than when emphasis was being placed on the work.

"Leaving off the crutch was an inspiration to nine tenths of the students. When I said, 'You are smart if you can leave it off, but if you need it you may use it,' most of them were delighted with the idea as a sign of progress. Those who depended upon it would erase it after using it for a day or two; then they found out how to get along without it."

#### *EAR-Section—Teacher C*

Lesson I. "Introduced steps in borrowing. A few seemed to understand perfectly. Many needed prompting. Could do Step 3 easier than the other steps."<sup>2</sup>

Lesson IV. "Found about five had forgotten how. About six were leaving off crutch, but answers were correct."

Lesson VIII. "All seem to understand. All using Step 3 only and some are leaving crutch off."

Lesson XIII. "Seem to understand what they do. A few try to add tens where it isn't necessary."

Postexperimental Period. "When the children had subtraction examples during the postexperimental period, some would have difficulty with them."

#### *EAR-Section—Teacher D*

Lesson II. "Difficult for students to understand application of principle to borrowing in subtraction so used. Used preliminary steps and it helped them to see it better."

Lesson IV. "Reviewed Step 1 and went on to Step 2. One pupil seemed to understand and tried to help the others."

Lesson V. "Two children discarded crutch at board but used it on paper. The girl explained her work, but the boy was confused as to an explanation for what he had done."

Postexperimental note. "In the tests given the children seem to be unable to drop the crutch. They do the ones' column but forget to add the ten in the tens' column. Every time subtraction examples are given some of them make this mistake."

#### *EAR-Section—Teacher E*

Lesson I. "The entire period was spent today on the new principle. At the end of the period I found that about five children could put in words what we had been doing during the work period."

<sup>2</sup> The unusual haste with which this teacher introduced Step 3 deserves comment. This shortened verbal pattern with the crutch was seldom presented by other R-teachers before the fifth lesson. In effect, Teacher C reports that she found the development of the principle hard to teach and passed quickly over Steps 1 and 2 to Step 3. Except for the crutch figures employed, Step 3 is similar to the verbal formulas used by the M-teachers. Such evidence as this tends to support the hypothesis of minimal differences in the experiment as formulated in chap. iii.

Lesson II. "I used Step 1 on the blackboard and after I explained and worked out a few examples with the class some of them could tell what we had done to get the right answer and why we had done it. I am hoping that they will all get it by tomorrow."

Lesson III. "I found that all but six children used the crutch correctly and got the right answers."

Lesson V. "We did not use Step 3 today as suggested, for I didn't find the children quite ready for it. I believe we can begin our change to Step 3 tomorrow. In the work today I was somewhat discouraged. There were many errors. They were all using the crutch. In the first number they were using it correctly, but in the second number they were making many errors."

Lesson VI. Used the short form. Noticed many mistakes.

Lesson VII. "Most of the children are using the verbal pattern (Step 3) which is shorter. I find that a few are not adding the ten to the second number, e.g.,  
They were not doing this until recently."

$$\begin{array}{r} 6\ 13 \\ 2\ 6 \\ \hline 4\ 7 \end{array}$$

Lesson VIII. "I noticed today, however, that many were not using the crutch in the second number, e.g.,  
Most of the class are using Step 3."

$$\begin{array}{r} 9\ 15 \\ 2\ 8 \end{array}$$

Lesson IX. "Today I checked each child individually and I found that only two children were using Step 2. They are both slow children but are always hard workers. I feel that they will be using Step 3 by tomorrow or the next day. We had more explanation and drill today on using the crutch in the second numbers. They understand that they must use it, but some few (about six children) are still leaving it off and therefore getting the wrong answers in many of their examples."

Lesson X. "They are using the crutch very effectively."

Lesson XI. "On eight of the papers the children failed to use the crutch in the second number. The other twenty-three children are using the crutch very effectively and have greatly increased their speed in subtracting with borrowing."

Postexperimental Period. "I have given them some practice sets to see how well they could get along without using the crutch. A few of the children made errors they hadn't been making previously. I feel that eventually they will get accustomed to working the examples correctly without using the crutch." (The extra practice was of course not legitimate.)

A comparison of the reports of the EAR-teachers with those of DR-teachers certainly leaves the impression that the principles of equal additions are harder to learn with understanding than is the principle of decomposition. The rationale for decomposition employs explanations which have experiential counterparts in the lives of third-grade children, whereas equal additions must be interpreted in terms of principles entirely new to them. This observation is borne

out by the length of time required to teach the principles of equal additions. Even where these principles were understood, it was hard for the children to see their application in the adjustments required for borrowing in subtraction. Other things being equal, the lower the intelligence of the pupils, the more difficult that application became. It follows, therefore, that the slower children—the very ones who needed additional learning aids—profited least from the EA-procedure.

*Summary.* Taken together, the diary records for the R-sections have abundantly justified earlier conclusions concerning the need for lower levels of performance. Many statements by the teachers pointed to the mental security provided by the longer patterns of thinking. Anecdotes were reported to show the increase in errors when a lower response pattern was abandoned too quickly. Finally, much evidence revealed that the crutches were easily dropped when learners had gained full control over the longer borrowing algorisms. Especially interesting was the testimony revealing the wide variation in the length of time the subtraction crutches were needed. Marked differences were disclosed from class to class and for individuals within any given class.

*V. To what extent did the various procedures employed measure up to the teachers' demands for an effective method for teaching borrowing?*

Only five of the fifteen teachers expressed a direct opinion concerning their reactions to their instructional program. In a number of cases satisfaction or dissatisfaction was implied in the daily comments, but only the explicit recommendations of the teachers have been recorded here.

#### *DR-Section—Teacher A*

Final comment: "They [the children] have come through better than I expected. I am sold on the DR-method. I have used it for years."

#### *EAR-Section—Teacher E*

"I do not intend to teach the EAR-method in years to come, for I believe that decomposition is better. It is more logical and easier to teach. The children seem to learn the decomposition method quicker and understand it better."

#### *DM-Section—Teacher A*

Postexperimental Period. "Most of the children seem to have no better understanding of the process. I believe this method of subtraction decidedly inferior."

*EAM-Section—Teacher C*

"Pupils and I have enjoyed this experiment very much. I think it is the easiest way to teach borrowing in subtraction. I am keeping all the material and hope to follow these same steps another year."

The comments just reviewed invite two observations. (1) The unanimous rejection of the DM-procedure by the teachers who turned in diaries agrees closely with the findings from the quantitative data presented earlier. (2) Of the three votes *for* a given procedure two were for DR and one for EAM. This proportion compares very favorably both with the results obtained in Center A and with the quantitative data for Center C. In general, it would seem that rationalization with decomposition may be used profitably even where a rather small amount of number meanings have been taught prior to the introduction of borrowing. The enthusiasm of the one EAM-teacher for her instructional program must be accepted as a fact, though it is inexplicable from the notes in her diary.

## EXPERIMENTAL CENTER B

The widely differing experiential backgrounds of the children in Centers A and C made it experimentally desirable to organize the diary comments for the two centers around the same general questions. Since the experiential background of the children in Center B was considered intermediate in meaningfulness to those in the other centers, it was felt that further investigation of the same problems in Center B would add little to the data already assembled. Likewise, a quick survey of the diaries for Center B revealed that the data describing the place of the crutches in borrowing agreed in almost every particular with those already presented. On the other hand, the question of teaching satisfactions derived from the use of the various procedures ("teachability") was not explored with any degree of thoroughness in the first two sets of diaries. The preliminary investigations of this question produced generalizations without much consideration of the evidence upon which those generalizations were based. Accordingly, the diary records for Center B were studied to obtain a more complete analysis of the teachers' reactions to their various instructional programs.

One of the advantages often claimed for mechanical procedures is that they are easy to teach. It is argued that efficiency in the use of the performance pattern may be established with greater rapidity and with less mental confusion than otherwise because the instruction is

not complicated by rationalizations. In this experiment some of these claims have been shown to be greatly exaggerated. The mechanical procedures produced less competence than did the rationalized procedures. Such has been one contribution of the quantitative data.

Diary reports that shed light upon ease of teaching and upon the presence or absence of mental confusion are now to be considered. In the earlier treatment of the "teachability" of the various procedures the data used were taken solely from the final evaluative comments of the teachers. The treatment here contemplated will review the issue more thoroughly by dipping into the teachers' day-by-day comments. All the materials used have been taken from Center B, although the content may be considered equally representative for the other two centers.

*I. Evidence concerning the relative ease of teaching D or EA in the M-sections.*

*EAM-Section—Teacher A*

Lesson I. "Lesson procedure for the first day was followed. The children seemed to understand the pattern. No questions were asked. The general attitude of the children toward the lesson was one of happiness at learning something new."

Lesson II. "There are no questions coming from the class. I use the verbal pattern and they accept it; then they attempt to do the seatwork. [On this particular day thirteen scores were above 70 per cent and seven were below that level.] The mistakes seem to be the result of the following:

1. Subtracting the top number from the bottom number.
2. Failure to add 1 to the tens' figure.
3. Poor command of the subtraction combinations."

Lesson III. "Care was exercised in the demonstration of examples. The seatwork was assigned, but the results were most disappointing. If 70 per cent is used as the passing mark, ten passed and seventeen failed." (Fifteen children had errors because of failure to increase the subtrahend.)

Lesson IV. "Miss C., our co-ordinator, was scheduled to teach the class as a demonstration for me, and so I reviewed in the early morning, using Set 3. Several examples were worked orally and then seatwork was given." (Note the extra, unauthorized practice.)

"Miss C. sent every child to the board to work an example. After that the examples were assigned for seatwork. As usual, the results of the tabulation showed fourteen failures and sixteen passing marks. The examples which presented the greatest difficulties were

|             |             |             |             |
|-------------|-------------|-------------|-------------|
| \$1.24      | \$1.56      | \$1.50      | \$9.98      |
| <u>-.93</u> | <u>-.63</u> | <u>-.60</u> | <u>-.08</u> |

These were the ones most uniformly missed. There were a great many others. The daily results are very discouraging to the teacher. Arithmetic is requiring too much time!" (Again, the evidence is that much more of the day was being devoted to arithmetic than the plan called for.)

Lesson V. "The day was used entirely as a study day. Correction of the most common errors for the fourth day's work was emphasized."

Lesson VI. "Problems were assigned after a discussion period. The results on these pages were *discouraging!* This method is certainly hard on the teacher. In this lesson seven passed and twenty-five failed."

Lesson VII. "After yesterday's results each problem was reviewed and worked in class. Emphasis was placed on how to reason when to add or subtract. Then we turned the page and read each problem, saying whether we would add or subtract to find the answer. The five problems were then assigned as seatwork along with fifteen examples to be copied from the board. On this day's work fourteen passed and seventeen failed."

Lesson VIII. "For early morning review work the class worked the examples in Practice Set 6. As a further review we read and worked the problems on page 90. After that the problems that appear on page 91 were read. We discussed each one and placed emphasis upon the larger number being on top and the smaller number being subtracted from it. After this study period the class was assigned the problem and examples for seatwork. [Scores showed twenty passing marks; twelve failed.] A little girl asked her mother why she had so much arithmetic in the third grade. Teacher and pupils are getting an overdose of arithmetic with poor results." (Here again, excessively long periods of arithmetic.)

Lesson IX. After preparation similar to that used for Lesson VIII seatwork was assigned. According to the teacher's scoring eighteen children failed and eleven passed. She found twenty-two children who had made one or more mistakes in the application of the formula.

Lesson XI. "The problems on pages 94 and 95 were studied in class. Each problem was read and then a child would tell how it should be worked. He told whether to add or subtract, which number was larger, which number we would subtract from which number, and what the names of the numbers should be. We spent so much time and emphasis on the analysis of these problems that it was very discouraging to have twenty children fail and fourteen pass. I don't think that I am that poor a teacher. I seem to have an average class and we have encountered no such difficulties before as we are experiencing in this EAM-method of subtraction."

Lesson XIV. "By way of review a number of examples were selected at random from the practice sets to be used as demonstration examples. These examples were placed on the board and the children worked them verbally as they were called upon. Then the teacher worked several. The children were asked whether anyone would like one explained again. Several hands were raised, and so the formula was repeated through several more examples. The scores for the papers showed twenty-three passing and eight failing."

Lesson XV. "I explained to the children the type of mistakes that they made in their work on the preceding day. A few demonstration examples were worked. Then the exercises on page 100 were assigned as seatwork. It took D. C. and D. H. two hours, and F. S. had to have two and one-half hours. The papers were scored; twenty-three passed and eight failed. Twelve of the children were still making mistakes in the application of the formula."

### *EAM-Section—Teacher B*

Lesson I. "Had splendid attention today in presenting the arithmetic. As usual, the children were anxious to go into something new. They quickly learned the verbal pattern. No questions were asked, just a comment from J. D., pointing to the board, 'You go up 1 every time on the top number and on the bottom number.'"

"After demonstrating an example we decided to work one or two examples before going into the seatwork. J. M. informed us that she missed the two examples because she thought she would save time by writing 'that bottom number' as she should see it, but she forgot and lowered the number instead of raising it. Eighteen children missed less than five examples and thirteen missed seven or more. Class worked slowly."

Lesson II. "Began lesson with extra practice from Lesson I. No question and no comments. Pattern, used verbally, went well. We spent so much time on our work today that we took up the papers and will have to complete tomorrow. We are spending a lot of time for such poor results." (Teacher considered papers of seventeen children satisfactory and those for thirteen unsatisfactory.)

Lesson III. "I am terribly distressed with results from Lessons I and II. The class used the shorter formula nicely when given orally. No questions yet. I'm surprised." (Fourteen papers considered satisfactory; sixteen unsatisfactory.)

Lesson IV. "Miss C., our supervisor, taught the class for me today at my request. I am worried over spending so much time on arithmetic and having such a large number fail. Miss C. felt that the class was getting along all right." (Here, as for Lesson II, and for Lesson VII below, the teacher admits allowing excessive amounts of time for arithmetic.)

"No questions asked. The class was not able to finish seatwork, and so will have to finish Monday. [Half of the thirty papers were considered satisfactory.] A feeling of relief at least after seeing records of the lesson I didn't teach!"

Lesson VI. "Today we studied together and worked the problems on page 88. I was sure of wonderful results from the seatwork on page 89 after so much discussion and study on page 88. But, alas— [Eleven of thirty-one papers considered satisfactory by teacher.] Only one conclusion: I am no 'fittin' teacher!"

Lesson VII. "As we hadn't gotten to Extra Practice Set 5, we used

that today. Also got in individual help and work on combinations. It would be needless to ask if we did anything else besides arithmetic."

Lesson VIII. "No questions; no comments; only a *few* right answers. I have never worked harder in my teaching experience and accomplished so little." (Eighteen children had one or more mistakes because of incorrect application of the formula.)

Lesson IX. "A comment from L ——— at least broke the monotony. While I was going over the formula with the pupils that had missed it she said, 'It's just like when you have two children. You don't buy something for one without buying something for the other one, too!'" (Thirteen papers acceptable.)

Lesson X. "Was *determined* to have better results, and so after going over our problems of yesterday, I prepared sixteen examples, similar to those, which could be used for seatwork. I also used them for demonstration purposes. Well, here are the results: Twenty-one of thirty-four papers considered satisfactory."

Lesson XII. Twenty-seven satisfactory, three unsatisfactory papers. Seventeen children made one or more errors in the application of the formula.

For the last three lessons the number of satisfactory papers ranged from twenty-three to twenty-six. On the last day only four of the papers were considered unacceptable.

Limitations of space do not permit as full treatment for Teacher C as has been accorded the first two. However, it may be said that the general tenor of her daily comments closely agrees with that of the comments of Teachers A and B. This same spirit is echoed in her final statement: "There were many times when I felt badly discouraged. Even now I realize that several of my pupils have very little idea of what it is all about."

At this point a statement needs to be made concerning the reasons for such an extended treatment of the EAM-diaries at the expense of the DM-classes. The writers feel that the more complete treatment of EAM is justified because in the past so many of the comparisons, based upon quantitative data, have shown EAM to be reliably superior to DM. This is to say that when in research studies borrowing has been taught mechanically, EAM has regularly earned the advantages in the experimental evidence presented thus far. On the other hand, DM has been so thoroughly discredited both by the quantitative data and by the diary reports from Centers A and C that only a summary treatment will be provided here.

Of the two teachers reporting experiences with the DM-procedure, Teacher B reveals a pattern of experiences very similar to that described by the EAM-teachers. At the time of Lesson XI twelve

children were still making errors in the application of the formula. In the summary statement she wrote:

The experiment has been very interesting, but my group of children need a rational method of teaching with complete explanations. Many of them do not subtract well after mechanical instruction. This method seems unsatisfactory, especially to the slower pupils.

DM-Teacher A also found that the slower children were the least able to apply the formula accurately and that their errors were hard to eliminate. By the time of the last lesson, however, most of her pupils were maintaining excellent records for accuracy. There were no postexperimental notes. The final statement of the teacher:

This was entirely too much arithmetic at one time for the children to take in. I do not like the method, but I found it very interesting. The children as a whole did much better than I expected.

*Summary.* All of the teachers in the M-sections of Center B found mechanical procedures undesirable from the viewpoint of personal preference. All but one reported results inferior to those habitually experienced. Many complained of difficulty in teaching, of slow progress, and of uninteresting and excessively long lessons. The errors of the children were found to be unsystematic in occurrence, and slow children were judged to suffer most by the mechanical procedures.

The fact that many of these criticisms of M-procedures originate from teachers who have been using R-procedures in their regular classroom teaching is of especial interest. If it be assumed, as claimed, that R-procedures are sources of mental confusion and that they slow up the rapid establishment of an effective subtraction performance, then these teachers should have been the very first to appreciate the benefits thus derived.

\* \* \* \* \*

A second question, discussed earlier, also needs further treatment. If it may be assumed that a rationalized procedure is to be used, is it more desirable (from the standpoint of the teacher) to use D or EA? Earlier evidence pointed to the conclusion that EA is more difficult to teach, but the validity of that conclusion is based on opinions obtained from the EAR-diaries in Center C. Perhaps different opinions might be held by teachers of children who had a more meaningful background of number experiences. To answer this

question additional data were collected from the diaries of the teachers in Center B.

## *II. Evidence concerning the relative ease of teaching D or EA in the R-sections.*

One very complete diary was submitted by an EAR-teacher who had taught DR in the preceding year. This teacher had asked to teach EAR in this experiment, and all the quantitative data available indicate that she taught it very successfully. It is especially helpful, therefore, to have the rather full account of her experiences in teaching EAR and her comparison of that procedure with DR. As much of her diary is reproduced below as space will permit.

### *EAR-Section—Teacher C*

Lesson I. "I worked entirely on the principle that the difference between two numbers is not changed if a ten is added to both numbers. I checked each child's work and questioned the class to see if they understood the principle. They seemed to understand it, but I did not find anyone able to word it very clearly. They seemed to understand the principle as applied to the thing they were doing.

"I tried working on through the twenties and thirties (extending the principle to higher-decade numbers) but that seemed to confuse them, and so I dropped back to the tens.

"I tried again to get from them a statement of the principle in their own words. They all talked very glibly about it, but the wording was vague. However, I think I got the idea across. Tomorrow I will review this before I take the next step."

Lesson II. "After reviewing yesterday's work and making another attempt to get a clear statement of the principle, I presented this example on the board  $\begin{array}{r} 65 \\ -28 \end{array}$ . The impossibility of subtracting 8 from 5 was readily seen. Their first idea was to subtract 5 from 8, but I explained that would be changing the example and we had no right to do that.

"When they suggested no other method I reminded them of the principle they learned yesterday, and asked if they could use that in an example like this. This suggestion was immediately accepted as a way to meet this difficulty, and they very readily added 10 to 5 and subtracted 8 from 15. But when they were ready to subtract 2 from 6 in the tens' column, nobody thought of adding a ten to the 2 before subtracting. I was doing the work on the board while they told me what to do, and so I did not subtract the 2 from the 6 but tried to get them to see that if they added a ten to the ones' column they must add a ten to the tens' column too.

"I first suggested that they were doing only half of the thing they had learned the day before. I went over yesterday's work again and

tried to tie the familiar fact to a new application, and then they seemed to realize that this was an application in adding a ten to both numbers.

"No work was done on paper as it seemed best to take the entire period today learning to understand the application of the principle and going through the explanation. Tomorrow we will take the examples on page 83."

Lesson III. "Today we went over some of the examples done on the board yesterday; then the class did the work on page 83. The same three children had difficulty with the work. Their common mistake was their failure to add a ten to the tens' column. The other children made practically no mistakes. We worked slowly, and I checked each one carefully and found the work very accurate.

"All the children used the crutch. It was no effort to get them to do it as all their explanations had required it. We used Step 2 in all the work today."

Lesson IV. "We talked over the mistakes made yesterday. They know their common mistake was their failure to add a ten to the tens' column. Then we discussed the *necessity* for adding a ten to *both* columns. Before we started work on page 85 I worked on the board examples number 1, 6, and 11.

"They did the work on page 85 quite accurately, but I think another day spent on two-figure numbers would have been better. It seems to me that it was rushing them to go into three-figure examples so soon. I am running one day behind the schedule now, and if I had it to do over again, I would take another day on two-figure numbers, followed by simple problems that would require application of what they had learned.

"We used Step 2 in all explanations today. The children needed it to clarify their thinking."

Lesson V. "I started the lesson with Step 2, which the children did beautifully. They did the work and explained Step 2 with ease.

"Then I explained several examples on page 86. I assigned the page for seatwork. This was the first lesson in which the class has disappointed me; and yet I can't say that the result was unexpected. I realized when I explained examples 6, 11, 16, and 22 that each of these examples was a different type and that the children were not familiar with any of them. The assignment contained too many new types for the average class to assimilate in such short order. They needed more time and practice on each type. I had only six perfect papers. On other assignments I have had as many as twenty or twenty-five perfect papers. The mistakes were like these:

$$\begin{array}{r} 40 \\ 31 \\ \hline 19 \end{array} \quad \begin{array}{r} 40 \\ 31 \\ \hline 11 \end{array}$$

Even yet the majority of the mistakes are like the first example.

"I won't attempt page 87 tomorrow. I will work on Set 3 until they understand the zeros better. It seems that we are attempting to crowd in too many steps in too short a period of time. I would spend much more time on each type of zero before going on to the next type."

Lesson VI. "I brought back yesterday's papers and gave a great deal of time to explaining the errors *collectively* and individually. Then we did Set 3, which turned out quite well. So far I have required Step 2 in all explanations."

Lesson VII. Page 87 assigned, collected, and sent to Duke University.

Lesson VIII. "We did page 88 on the board, first discussing [the problems]; then I assigned page 89. The problems were more difficult than those my children have done in their workbooks, and so they had considerable difficulty with them. They also made mistakes in subtraction. They always have difficulty with this type of example,  $\overset{34}{-9}$ . In this

example,  $\overset{74}{-58}$ , about half the class forgot to add the ten to the tens' column. I had to remind them of the principle they had learned. The problems made a new application, and it did not carry over.

"All the children use the crutch every day; it never seems to occur to them that the work could be done without it."

Lesson IX. "Page 90 was assigned for seatwork after I had called the class's attention to mistakes made on the preceding lesson. This time the work was done with few errors."

(During the second part of the lesson another practice set was assigned. The thirty children made a total of twenty-nine errors—an average of slightly less than one error per child.)

"Of the twenty-nine errors made, twenty were failures to add a ten to the tens' column. The others were mistakes in the subtraction facts.

"Every day before we begin work we review the principle of adding a ten to both columns. I find less tendency to turn the example upside down than I have found in former years when teaching the decomposition method, but I have equal difficulty getting established the habit of adding a ten to the tens' column."

Lesson X. Page 91 was assigned without preliminary discussion of problems. Only two papers had mistakes in subtraction. Errors of one child were attributed to four days of absence, but the other pupil had been receiving special help every day. Twelve additional examples were solved with twenty of the thirty children making no more than one error. Most of the above mistakes were failures to add a ten to the tens' column.

Lesson XI. "Discussed the problems on pages 92 and 93. Gave special help with problem 4, which is too hard for third-grade children. They did the others well, and remembered to apply the principle of subtraction they have learned."

Lesson XII. "Today I gave the problems on pages 94 and 95. We discussed them before the children did the work. The work was excellently done with few mistakes. J. B. is still making mistakes in method and facts. Later, I gave Set 9, and it, too, was well done.

"I haven't mentioned the crutch in this log for some time, but that is because it has been no problem. The children all use it every day. I never have to mention it to the class."

Lesson XIII. The first three rows on page 95 were assigned for first part of the lesson. Two children having much difficulty showed no improvement.

"Later I assigned the last three rows on page 95, collected the papers and listed the mistakes as follows:

14 children made no mistakes

5 children made mistakes in subtraction facts

4 children forgot to add a ten to the tens' column

6 children subtracted minuend from subtrahend

3 children did not finish; mistakes not listed."

Lesson XIV. "The children continue to use the crutch with no reference to it being made by me. It hasn't seemed to occur to them that it could be discarded."

Lesson XV. Work on page 98 assigned, papers collected and sent to Duke University.

Postexperimental Period: Jan. 4. "Today we reviewed subtraction, discussed the crutch and how we could do without it now that we know how to subtract. They did some examples and I reminded them not to use the crutch. They discarded it with apparent ease and the accuracy of the work was not affected."

Jan. 6. "I put eight examples on the board today with instructions not to use the crutch, then said no more about it. Two children were the only ones to use it. They did the work over without the crutch.

"Results ran true to form; the ones who usually have perfect papers had perfect papers today and the ones who made the most mistakes with the crutch made mistakes without the crutch. The common mistake was the failure to add a ten to the tens' column. This has been the common error all along. No child whose work has been outstandingly good seemed at all affected by the loss of the crutch."

Jan. 18. "Out of an activity today there came a number of problems which required subtraction. I was gratified at the accuracy with which they were done. No one used the crutch."

Jan. 26 and 27. "No one ever uses the crutch. The children have not used it even once since the second day after we discarded it.

"I gave twelve examples today and checked the mistakes. Twenty-two children had no errors, seven made but one error, and one child had three errors. Of all the mistakes made only three were mistakes in method; the others were mistakes in facts."

Evaluation of the experiment. "All the way through the experiment I found the assignments too long, particularly at first. I took more time on the first three lessons (the flexibility of the program permitted this, so that you will find that the third experimental day was really my fourth assignment). All the way through I was one day behind the assignments. Also I required the children in explaining their work to give Step 2 longer than the directions suggested, but I found that necessary when difficulties arose, and with slow children I frequently went back to it all during the experiment.

"My children had considerable work with ones and tens before the experiment, but I feel that if the experiment had come later when they had acquired a better understanding of units and tens, they could have grasped the meaning of the EAR-method more easily. They learned the words of the formula of explanation, but I feel that it lacked meaning to them, and more meaning could have been acquired if they had had more time and practice with units and tens.

"However, as I attempt to evaluate the experiment the most outstanding thought in my mind is that the whole experience lacked meaning to the children. The attempt to rationalize the EAR-method was a most difficult job; and even though we went through the steps with objects, giving concrete examples of the principle, the situation seemed an artificial one. The children had no everyday experiences that contributed to a better understanding of that principle; consequently, it lacked background.

"At first the slow children were confused by the method, because they both added and subtracted in the same example, but individual help and practice seemed to clear that up. Not remembering to add a ten to the tens' column was the most persistent error in my class, and it seems that only practice will establish the correct habit. In this type of example

the children had great difficulty at first,  $\overset{126}{-84}$ , but towards the last I noticed improvement.

"I volunteered to teach the EAR-method with a thoroughly open mind. My choice was made for the express purpose of finding an easier way of teaching subtraction than the decomposition method, *if that method existed*. All the way through the experiment I followed directions, carefully checked the work, attempted to analyze all mistakes, and continually retaught the principle giving trouble. I did not find that the EAR-method had any advantage over the decomposition method in speed, accuracy, or understanding. Personally, though, I am delighted to have had this opportunity to make the experiment."

In the diary account just summarized Teacher C has clearly pictured the difficulty experienced in getting the principles of EA to function effectively. The diary of EAR-Teacher B reveals even more difficulty in this respect during the first three days:

#### *EAR-Section—Teacher B*

Lesson I. The teacher spent the entire lesson on the mathematical principle. She felt that the class was ready for Step 1 by the end of the period.

Lesson II. Reviewed the mathematical principle, adding tens with money, objects, and in verbal problems. When the application was made to Step 1, however, difficulties were encountered. "Many children added a ten anywhere: to the tens in the minuend as well as to the tens in the subtrahend. In an example, such as  $37 - 19$ , they thought 11 tens just

as they did 17 ones." (Thirteen children made such errors.)

Lesson III. "Lesson plan same as second day with great improvement on the understanding and working of Step 1."

The diary for EAR-Teacher A was not very complete, but she spoke of spending the whole period on the first day on Step 1. If so, it is unlikely that the process of borrowing was given much attention in this class.

Portions of two of the DR-diaries are reproduced below to facilitate comparison of the ease with which the two different subtraction procedures were taught.

#### *DR-Section—Teacher C*

Lesson I. "Since we had already had experience in the preinstructional period in analyzing two- and three-place numbers, Step 1 was not difficult for the class to understand. The class suggested borrowing from the tens' column. With a few of the slower members of the class we had to resort to sticks (in bundles of ten) on the first day. One child asked if we ever had to borrow two tens. This question was answered and explained by the class. A number of the children worked examples at the board using the crutch. The verbal pattern came a little slowly and hesitatingly, but I could see there was understanding. After working with individuals at the board, I gave three examples from page 82 as a quick test. Seven members of the class failed to make a perfect score."

Lesson II. "I presented a few 'number pictures' on the board at the beginning of the lesson. I gave individuals an opportunity to work at the board. The children seemed to have very good understanding. Most of the errors were due to the fact that many pupils did not know the subtraction facts.

"With the exception of one boy the class consistently used the crutch. After I had worked with him individually he seemed to understand it. I was pleased with the results for the second day."

Lesson III. "I demonstrated on the board five examples from page 85. I did not think it necessary to go back to Step 1. When I assigned the rest of the examples on the page, all the pupils used the crutch except one who had been absent on the second day. I'm convinced this is the best method—especially for the slow group I have."

#### *DR-Section—Teacher A*

Lesson I. "We started out with Preliminary Step b. This meaning of numbers had been developed since the beginning of school. Then we went to Step 1, which gave no trouble in Parts One and Two. We worked out the answer in Part Two (of Step 1). When we went to Part Three a good many were ready to suggest that we use one of the tens. Then we started working with Step 2.

"Most of my children have fairly good concepts of numbers, and the

work from this point on moved along nicely. We had time for them to work on paper only the four examples under Step 2."

Lesson II. "We used Step 1 with two examples and then went directly to Step 2. We worked a good many examples on the board, developing the verbal pattern all the while. Two children were still confused, thinking that the borrowed ten added only 1 to the ones instead of 10 ones.

"When page 82 was assigned the children worked well. Twenty of the twenty-three present did creditable work.

"One common error was the placing of the crutch numbers beside the tens and ones instead of above and to the side."

Lesson III. "We used only Step 2 and the children seemed to 'catch on' readily. There are still three little girls who don't know what it's all about. When the examples on page 85 were assigned most of the errors seemed to be caused by:

1. Failure to copy correctly.
2. Not knowing the subtraction facts.
3. Failure to draw the 'number picture' in a few cases.
4. Crowding of work on the paper."

The diary notes for the remaining lessons disclose that a number of difficulties were encountered by both DR-classes before the children learned to borrow efficiently; but rarely did these comments indicate a failure to understand the meaning of D or how it was used in the process of subtraction. In this respect it won the approval of the teachers over EA.

To conclude, it needs to be emphasized that the extended treatment of the diary materials in this chapter has been presented, not in lieu of more objective data, but rather as an experimental confirmation of the quantitative results reported earlier. It seems reasonable to believe that when quantitative results are supported by personal observation, they have been doubly validated.

## CHAPTER VII

### CONCLUDING STATEMENT

In view of the abundance of rhetoric in modern education on the importance of meaningful experience in learning, the dearth of competent relevant research stands in strange contrast. In both oral and written discussion all sorts of claims are advanced for instructional programs based upon meaningful experience; but few indeed are the research studies to support these claims. No subject matter area on the curriculum, elementary and secondary alike, is exempt from this anomalous state of affairs.

The discrepancy between claims on the one hand and evidence on the other is particularly striking in the case of arithmetic. Almost without exception students of arithmetic are now agreed that the subject must be understood, that learning which is not grounded in understanding is pseudo learning, not worth the time and effort required to achieve it. Advocates of meaningful arithmetic (the writers included) argue that learning is more economical, that ideas and skill are better retained, and that these products of learning are more available for useful transfer when children see sense in the arithmetic they study.<sup>1</sup>

Such are the arguments for meaningful arithmetic. What evidence is there to support the argument? One bit of evidence is deduced from the meager results produced by the arithmetic taught in the quarter-century ending about 1940. In spite of six to eight years of school instruction in arithmetic, our adult population is in general quantitatively unintelligent. (Witness, for example, the examination records made by enlisted men and women in the armed forces.) Now, in the period in question the arithmetic program was characterized by emphasis upon mechanical skills with correspondingly little attention to understanding. It is reasonable to suppose that more stress upon understanding might yield more commendable results. A second bit of evidence is inferred from psychological experimentation on learning. (See Chapter I, pp. 20-21.) This research has unequivocally demonstrated that learning is economical

<sup>1</sup> For an elaboration of these claims, see William A. Brownell, "The Place of Meaning in the Teaching of Arithmetic," *Elementary School Journal*, XLVII (1947), 256-265.

to the degree that learning tasks are meaningful to experimental subjects. It is to be supposed that increases in meaningful instruction in arithmetic may have corresponding effects. A third bit of evidence is provided by practical experience with meaningfully taught arithmetic. Here and there are school systems, or particular schools or particular classes, where the attempt to invest arithmetic with meaning has proved to be profitable. A fourth bit of evidence—and it is scarcely more than that—is to be found in a relatively few research reports which bear directly upon the issues involved in teaching arithmetic with meaning.

Taken as a whole, this evidence for meaningful arithmetic is not very convincing. The first two types of evidence are admittedly inferential. The third is drawn from isolated instances where meaningful instruction has seemed to be successful. One must exercise caution, however, so as not to generalize too broadly and too optimistically from too few cases. The last type of evidence, the best of the four, is limited in its extent. By now, probably more than two thousand investigations relating to the teaching of arithmetic have been reported. Of this number not more than fifty or sixty bear even indirectly upon questions pertaining to the place of meaning in instruction.

It is highly improbable that any one investigation can *prove* the case for meaningful arithmetic. The question of the place of meaning in arithmetic instruction is too big and too complicated to be answered by a single definitive study, and any attempt to draw broad generalizations from such a study must almost certainly oversimplify both the issues involved and their final solution. As a matter of fact, some of the most ambitious studies, employing large populations and carried on for months according to carefully prepared experimental schedules, have come to grief, apparently defeated by the very size of the problems attacked. Certainly, promising as they seemed to be, they have failed to yield decisive evidence for meaningfully taught arithmetic.

Experimentally, the most hopeful approach seems to lie in a large number of less ambitious undertakings, less ambitious only in the sense that the problems selected for study are limited in scope and are clearly manageable through the application of appropriate research techniques. In some of these studies the period of experimentation may have to be kept relatively short, and the numbers of teachers and subjects may be unimpressively small. Extent may

have to be sacrificed for depth, in order to establish and maintain the requisite controls. Yet "sacrifice" is hardly an apt word, for the gains can be great. It is then possible to increase the number and the variety of observations; not only this but to make them more penetrating and intimate. In this connection the research of Wheat<sup>2</sup> and his students may be consulted to advantage.

In the present investigation an attempt was made to study a narrowly defined problem: Is it better to teach third-grade children to borrow in subtraction by decomposition or by equal additions, and to teach them by meaningful or by relatively meaningless programs of instruction? Had the writers foreseen the difficulties and complications which would arise<sup>3</sup> from setting up four independent experiments in as many cities and involving forty-one classes enrolling 1400 pupils—had they had this foreknowledge, they probably would have restricted their experimental populations as they did the experimental problem. Be that as it may, the purpose was the same in each experiment, namely, to secure as many kinds of crucial data as possible concerning the effects of teaching the two borrowing procedures by different methods. The details of the experimentation are omitted here, but they are fully set forth in Chapter II and in Appendix C.

The experimental findings have been reported, center by center, in Chapters III, IV, and V, together with the teachers' reports of their experiences in Chapter VI. As a whole, the findings show that meaningful instruction, especially in the case of decomposition, produced results superior to those produced by mechanical instruction. But that is as far as they go in *proving* the value of meaningful teaching.<sup>4</sup> They represent only a bit of new evidence, and their im-

<sup>2</sup> Harry G. Wheat, *Studies in Arithmetic: A Summary of Masters' Problems and Theses Dealing with the Subject of Arithmetic* (Morgantown, West Virginia: West Virginia University, 1945; 63 pp.).

<sup>3</sup> The difficulties of securing strict adherence to experimental directions have been alluded to several times in chaps. ii-vi. Fortunately, most of the deviations from the experimental plan were of no consequence, for they related to minor issues or to the supplementing of data which, in so far as the central problem is concerned, were already sufficiently varied and comprehensive. One weakness, however, could have had serious consequences, namely, the failure of some teachers to follow faithfully the prescribed programs of instruction. The consequences were not serious, however, for even with the irregularities in teaching the differences among the experimental factors revealed themselves unmistakably and consistently. The only effect was to make these differences *as here reported* much less than they certainly were, or should have been.

<sup>4</sup> That is to say, the findings do not in themselves make a complete case against mechanical and for meaningful instruction in arithmetic. They do, however, have significance for one aspect of the problem relating to the place of meaning in arithmetic instruction, an aspect which has been badly neglected. See point 3, p. 151.

plications for arithmetical instruction in general can properly be extended only to other phases of the subject which, in their psychological demands, are clearly analogous to the skill of borrowing in subtraction.

#### OBSERVATIONS WITH RESPECT TO THE DESIGN OF THE EXPERIMENT

As was explained in Chapter II, the present inquiry was planned in such a way as to correct what were regarded as the principal weaknesses in previous research on the central problem. The data summarized in the foregoing chapters demonstrate the reality of these supposed weaknesses and the wisdom of making changes in the design of the typical investigation of the comparative merits of EA and D.

1. The suspicion that in earlier studies the problem had been oversimplified has been amply demonstrated. In these earlier studies (see pp. 5-16) the aim was to establish once and for all, in absolute terms, the superiority either of EA or of D. The present investigation has shown the futility of any such attempt and the necessity of taking into account many factors which condition the advantages of the two procedures. Among these factors are: age of pupils (or, better, amount and kind of arithmetical backgrounds), methods of teaching the procedures, outcomes set in teaching the skill of borrowing. Hence the decision to teach D or to teach EA must be relative, and must be made with due consideration for these factors.

2. Unlike previous studies, with a few exceptions, the present inquiry was instituted in Grade 3, that is to say, at the time the subjects were introduced to the process of borrowing. The experimental findings in the present instance differ from those of previous research at some points. These differences may be well accounted for, in part at least, by the dissimilarities in the backgrounds of the experimental subjects and in their relative familiarity with borrowing as a subtraction skill. In accepting the conclusions for the present study *for Grade 3* one need not repudiate wholly the conclusions of other studies as they apply to later grades.<sup>5</sup> But one is not justified in holding that the conclusions of previous studies, perhaps valid for later grades, are equally valid for the initial stages of learning to borrow, as in Grade 3.

<sup>5</sup> As will be made clear later (point 4, p. 156) this sentence does not state the issue in the best way. The problem of "when" is not so much a matter of grade level as it is a matter of foundation experiences.

3. It was inferred (inferred because the experimental reports are incomplete at this point) that earlier studies called for the teaching of borrowing in a mechanical fashion, with little or no effort to make the process intelligible. In the present investigation the two borrowing procedures were taught mechanically in two sections in each center; but they were also taught rationally in two other sections in each center. The findings for the M-sections agree fairly well with those of previous research; but the findings for the R-sections validate the hypothesis that the method of teaching the procedures is an important (if neglected) factor in determining the worth of the two procedures.

4. The experimental results in the four centers are in essential agreement at nearly all major points, but there are some differences. These differences justify the decision to treat the data separately by centers. EAR made its best showing in Center A, and its poorest showing in Center C. Apparently the children in Center A had rich enough backgrounds of meaningful arithmetic to profit by the attempts to rationalize EA; but not so in Center C.

5. The separation of "smoothness of performance" from "degree of understanding" had important consequences. In the first place, it prevented interviewers from crediting to "degree of understanding" an aspect of performance which might otherwise have been confused therewith. In other words, it resulted in "purer" measures of understanding. In the second place, the separation demonstrated emphatically that "smoothness of performance" has no functional relationship with skill in borrowing. The M-sections, which merely mastered a verbal pattern given them, rather consistently outscored the R-sections in "smoothness of performance"; but they were superior in nothing more than glibness.

6. The measures of "degree of understanding" proved to be invaluable, more than repaying the extraordinary expenditures of time required to secure them. (The estimate of five hundred hours is almost certainly too small.) In the first place, these measures provided direct evidence on the success of the teachers of R-sections in investing EA and D with meaning, and not only on their success, but on their *relative* success. In the second place, they greatly aided in interpreting the greater competence of the R-sections on the Retention and the Transfer Tests.

7. No previous research on the comparative worth of EA and of D has been carried on long enough to secure measures of retention

or of transferability of the skill taught. Yet a moment's consideration discloses the essential character of these aspects of learning. Inclusion of these criteria of learning greatly broadens the base for evaluating the advantages of the four programs for teaching borrowing.

#### FINDINGS WITH RESPECT TO D AND EA

1. If the experiment had been restricted to the one investigation in Center A, the general conclusion would be: First, decide what outcomes you intend to achieve in teaching borrowing; then teach either D or EA accordingly. In other words, the primary and immediate question would not be, Shall we teach D or EA?, but rather, What kinds of knowledge and skill do we want to develop in teaching borrowing? For example, do we want mere proficiency in mechanical subtraction, a high order of accuracy and rate of computation? Or, in addition to competence in terms of accuracy and rate, do we want pupils to understand what they learn and to be able to extend their skills readily, and thus to lighten the learning and teaching required to master more advanced forms of borrowing?

In the light of the data for Centers B and C, it is clear that the conclusion mentioned above holds only in the case of children with relatively rich backgrounds of meaningful arithmetic (like those in Center A). Such children have some ability to work out their own rationalizations, though it would seem to be safer even for them to be given instruction at this point, especially when EA is the approved procedure. On the other hand, for children with limited backgrounds of meaningful arithmetic (as in Centers B and C), the choice between D and EA is different. Regardless of the criteria employed (except possibly for smoothness of performance and rate of computation), the choice is in favor of D, and that taught with meaning.

2. The EA-procedure is difficult to rationalize. Children like those in Center A can grasp the relationships and principles involved, but not so readily as they can do so with D. As for children like those in Centers B and C, the rationalization of EA seems to make impossible or at least excessive demands.

The testimony of EAR-teachers in these two centers is unmistakably in this direction. Even those who were most successful in rationalizing EA admitted extreme difficulty, complained of getting seriously behind in their experimental schedules (although an extra day was allowed for rationalizing EA as compared with D), and more than once resorted to teaching periods of extreme length.

The trouble seems to lie in the fact that EA involves a new mathematical principle which is foreign to the number experiences of third-grade children. This principle is that the difference between two numbers is not disturbed if the same amount is added to both terms. In its application this principle requires the alteration of both minuend and subtrahend. The change in the minuend is easy enough; but not so the change in the subtrahend. Day after day, the EAR-teachers state, their pupils altered the minuend correctly, but failed to alter the subtrahend.

These facts would seem to answer once and for all the question about the relative ease of rationalizing D and EA in Grade 3. When teachers of D report no difficulty in rationalizing that procedure, when teachers of EA report serious difficulty in rationalizing that procedure, and when teacher experience is corroborated by quantitative evidence of the same import, then the matter would appear to be settled. It may be recalled that Johnson (see p. 8), in claiming no difference between the two procedures from the standpoint of ease of rationalization, called attention to the lack of evidence on the issue. That evidence is now available.

3. Some proficiency can be produced by *mechanical* instruction with either D or EA. The present findings, save for Center C, are in favor of EA. In this conclusion the present research verifies the results of earlier studies. One must not, however, expect children with limited arithmetical backgrounds in meaning to *understand* their procedures; they will not devise their own explanations. This conclusion is confirmed well by the diary statements of DM- and EAM-teachers in Centers B and C. On this point previous research is silent, for it made no provision to secure measures of understanding.

#### FINDINGS WITH RESPECT TO THE CRUTCHES

The crutches taught the R-sections in the experiment are illustrated for the example  $362 - 129$ . These

devices were intended to serve as visual patterns for the procedures employed in borrowing, and were taught as parts of the rationalizations therefor as "number pictures."

| DR                                | EAR                               |
|-----------------------------------|-----------------------------------|
| $3 \overset{5}{6} \overset{1}{2}$ | $3 \overset{5}{6} \overset{1}{2}$ |
| $\underline{-1 \ 2 \ 9}$          | $\underline{1 \ 3 \ 2 \ 9}$       |

In the recent past, devices of these kinds, commonly called crutches, were looked upon with disfavor. Objections to them took such forms as: (a) they are unnecessary; (b) they are hard to learn; (c) they do not advance learning in the function to which they are

attached, and may even interfere with it; (d) once learned, they tend to persist, being very resistant to all attempts to dislodge them. Two earlier studies<sup>6</sup> of the D-crutch found little basis in fact for these objections. The evidence from the present investigation is similar in character.

1. The teachers' diary reports, as analyzed in Chapter VI, (pp. 135-146, and especially 124-133) demonstrate that the children, particularly in Centers B and C, actually needed the crutches (or similar aids) to assist them in their transition to abstract processes of borrowing.

2. The D-crutch was simple to learn. The teachers of DR-sections state that their pupils mastered it quickly and enjoyed changing minuends in borrowing examples. The EA-crutch seems to have been harder to learn, probably because it called for changes in both minuend and subtrahend, the latter change being a bit awkward to make in the limited space available. Still, the EAR-teachers' comments imply that the trouble lay less in using the crutch than in applying the new principle involved (addition to both terms).

3. That the D-crutch facilitated learning to borrow is to be inferred from the consistently good records made by DR-classes. The EA-crutch seems to have served a similar purpose in Center A, but not in the other two centers. Just how much good (or harm) the EA-crutch was responsible for is problematic for the reason mentioned in the immediately foregoing paragraph.

4. That the crutches *can* be discarded, and without too much trouble in the case of most children, is evident from the data in Figures 1 to 6. Used during the Experimental Period proper by practically all children in practically all borrowing examples, the crutches were pretty well gone by the end of the Postexperimental Period. True, some children (perhaps one in five or six) depended upon their devices more or less regularly, making excessive errors when they tried to borrow without them. It may well be that most of these children should have been allowed to use the crutches as long as they needed them—this of course on the assumption that they really did need them.

<sup>6</sup> William A. Brownell, *et al.*, *Learning as Re-organization* (Duke University Research Studies in Education, No. 3. Durham, North Carolina: Duke University Press, 1939); D. Banks Wilburn, "Learning to Use a Ten in Subtraction," *Elementary School Journal*, XLVII (1947), 461-468.

## FINDINGS WITH RESPECT TO THE EFFECTS OF UNDERSTANDING

1. One claim commonly made for understanding as a factor in learning is that its effects are cumulative. Evidence to support this claim is to be found at two points in the present study. (a) In Center A many children in M-sections undertook to invent their own explanations for D and EA when explanations were withheld by their teachers. (b) The forward effect of understanding is revealed in the relatively higher accuracy scores earned by R-sections in the Retention Test and in the Transfer Test.

2. It is no simple matter to build meaningful learning upon a foundation of mechanical learning. True, in Center C the teachers did succeed in teaching DR to children who were unaccustomed to thinking rationally about arithmetical skills; but the principles which underlie D are readily objectified and explained. On the other hand, when the teachers in Center C tried to teach EA rationally to comparable children, they were not at all well satisfied with their hard-won results. Indeed, certain ones of these teachers expressed doubt that some of their pupils ever really did understand the procedure, however apt they were in reciting the correct verbal pattern.

3. The term "meaningful arithmetic," as it has been (necessarily) used throughout this report, is misleading. It implies the existence of a single, generally accepted program for teaching arithmetic with a view to sensible learning. This implication is contrary to fact, as anyone can determine for himself by comparing the variety of programs sponsored by those who advocate "meaningful arithmetic." These programs differ materially (a) in the particular meanings (concepts, relationships, principles) which are selected for teaching and (b) in the relative stress put upon these selected meanings.

Arithmetical meanings are not all of the same order of importance for learning. Some are absolutely essential if arithmetic is regarded as a logical system of quantitative thinking. Others, according to this conception, take lower places in the hierarchy of importance.

The point of the foregoing paragraphs is well illustrated in the present study. Center A pupils had had instruction in the basic principles of the number system and of computational procedures. Place value had been emphasized, and carrying in addition had been explained in terms of ones, tens, and hundreds. With this background, most of the R-children in this center responded quickly to instruction in borrowing both by D and by EA, and many children

in M-classes tried for a time to find their own explanations. This condition obtained to a much less degree in Center B. True, some of these children had had more or less "meaningful arithmetic" prior to Grade 3, but the instruction was not general, systematic, or apparently directed toward the fundamental relationships. As a consequence, most of the pupils enrolled in the M-sections seem to have been able to make little use of such meanings as they had, and those enrolled in the EAR-section never did master the rationale of that procedure.

The term "meaningful arithmetic" is seriously in need of competent analysis. Research must discover and arrange in order of importance the arithmetical meanings which should be taught. Only when a program of "meaningful arithmetic" is defined in this way can it be known just how "meaningful" it is. When (or if) this analysis and this differentiation are undertaken, at the top of the list of essential meanings will be found those which govern our procedures in computation. Such is the opinion of the writers, at any rate, who see this probability in the data reported in the monograph.

4. There is another way in which the findings of this study are significant in the general question of the place of meaning in arithmetic instruction. It is still common for school people to think of borrowing in subtraction and other such skills as belonging in this or that grade, almost as if the time and place were set immutably by some kind of law. This notion is epitomized in such statements as that children are "ready" for the addition combinations in such-and-such a grade, for subtraction of two-place numbers without borrowing in such-and-such a grade, and the like. Elaborate paraphernalia of testing may be instituted to determine whether children in any particular group have attained the magic state of "readiness" for a skill or topic.

To the degree that these testing programs are sound, they turn out to be no more than inventories of the skills and knowledge requisite to the new learning. And, for this reason, they are valuable, save that they need to be supplemented by measures relating to essential concepts and understandings. But the point is that the tests in such programs expose "readiness" for what it is, namely, the ability to take on new skills or ideas, that and nothing more. If children have the necessary ability, knowledge, and understanding, they are "ready." And their "readiness" is determined, not by their age or by their grade, but by the kind of arithmetic they have had.

By manipulating their arithmetical backgrounds we may shift the placement of any arithmetical topic over a range of several years and of several grades.

Certainly the children in R-classes in Center C were not "ready" in Grade 3 for instruction in borrowing as a rational process. It is doubtful whether these children would have been "ready" for such instruction in Grade 4 or Grade 5, provided that they continued to be taught arithmetic as a bag of tricks. On the other hand, the Center A children, with their unlike backgrounds in meaningful arithmetic, were in Grade 3 entirely "ready" for rational explanations of D and even of EA. And the findings for Center C may be interpreted to mean that the R-children were "ready" in Grade 3 for D when taught rationally, but "unready" for EA when taught in this manner. To repeat a statement made earlier, the "time when" to teach an arithmetic topic is not a function of age or of grade, but a function of the kind and amount of previous learning.

#### FINDINGS WITH RESPECT TO ECONOMY IN LEARNING

That program of instruction which most quickly establishes mastery of a skill is frequently regarded as the most economical. In the skill of borrowing in subtraction, for example, what seems to be needed is facility and accuracy in computation; and the best (most economical) way to secure them is to work on them and on them alone, from the outset. In this view any attempt to make explicit the relationships inherent in borrowing is, at the best, wasteful. It may even be harmful, for it misdirects attention on the part of the learner and diverts him from the goal of proficient computation.

By common consent accuracy and speed are essential learning outcomes in the skill of borrowing; but the most economical route to these ends is not what appears to be the most direct route. Otherwise, the M-sections would have consistently outscored the R-sections in the series of experimental tests. Of course they did somewhat generally compute at a greater rate; but their inferiority in accuracy more than offset this advantage. After all, there is little value in being able to get incorrect answers faster than someone else.

According to the results of this investigation, the most economical route to speedy *and* accurate computation in borrowing examples is an indirect route. It is a route which includes excursions into the rational principles that govern borrowing. No other interpretation of the superiority of the R-sections in the three centers seems at all reasonable.

## IMPLICATIONS FOR THE TEACHING OF BORROWING

1. In the matter of teaching borrowing in subtraction the implication of the present study is that systematic attention to understanding pays dividends. Ease of teaching the skill rationally seems to be proportionate to the amount of understanding which the child brings to the skill. Moreover, the same relationship extends into the future. The more meaningful the first experiences with borrowing in the simpler types of example, the readier the learner is for more advanced types of borrowing.

2. To the extent that other skills in arithmetic are analogous to the skill of borrowing in their psychological demands, emphasis upon understanding in learning should have comparable results.

3. Borrowing is commonly taught in Grade 3 in American schools by the D-procedure. The findings of the present study vindicate this practice, but only when D is taught rationally. When D is taught unsuccessfully, it may be inferred that it is taught mechanically. If we are to continue to teach mechanically, we had perhaps better change to EA. But the bargain would be a poor one. If the results of this investigation mean anything, DR is much to be preferred to EAM.

4. One cannot close his eyes to the conclusions reached in previous research. In this earlier research, carried out for the most part in Grade 5 and above, EA has been consistently shown to be much faster than D, as well as being a little more accurate. Here there seems to be something of an impasse: the procedure which is better in the initial grade of learning is poorer in the later grades. What is to be done about the matter?<sup>7</sup>

If the impasse is a real one, several choices are open to us. First, we may disregard the new evidence in favor of DR in Grade 3 and teach EA from the start, on the ground that in the end the results will justify this decision. To do so, however, would be to deny children insights possible when borrowing is taught by DR; or, in a word, to say that understanding in learning is of no importance. Such a statement has been made more than once. See, for example, the quotation from Ballard, page 19 above. With this position the writers (and, they hope, most modern students of arithmetic) are in violent disagreement.

<sup>7</sup> Actually there may be no impasse at all. We have no convincing data to show what happens in later grades when D is taught rationally, or when EA is taught rationally, for that matter. The data available have all been secured, or nearly all, from classes taught mechanically. (Such at least is the inference to be drawn from experimental reports, save for a very few instances.)

A second choice is the reverse of the first: Teach DR, and DR only, from Grade 3 onward. This is the decision to which most American schools have come. (At least, they teach D earlier than M.) But in so doing, we lose sight of the greater efficiency of EA in the higher grades; or we say that this later advantage is of no consequence.

A third decision would be to teach DR in Grade 3 and to introduce EAR later, say in Grade 5. For this proposal it can be argued that in learning EAR Grade 5 children will acquire a valuable new mathematical principle, and that they will acquire it when they can really understand it. The principle, that the difference between two numbers is unaltered by the addition of the same amount to both terms—this principle is of wide application, and children ought to know it. Against the proposal it can be argued that the change in procedure might work havoc on children's skill in borrowing. Why teach one procedure, only to desert it later for another?

But would children experience much loss in skill if they changed in Grade 5 (or later) from D to EA? We have little evidence on the point as yet,<sup>8</sup> but we need it. Here is an excellent problem for research; moreover, a problem which could be easily investigated, since many schools can be found in which the children know nothing of EA, having used D from Grade 3 onward. Such a study would supply us with a basis in fact for adopting or rejecting the third alternative concerning the program of instruction best adapted to teach borrowing to a high degree of lasting proficiency.

<sup>8</sup> Winch reports that twelve-year-old girls, taught EA originally, then changed to D, after a short period of practice subtracted better by the new procedure than a parallel group which continued with EA. See W. H. Winch, "Equal Additions' versus 'Decomposition' in Teaching Subtraction: An Experimental Research," *Journal of Experimental Pedagogy*, V (1920), 207-220; VI (1921), 261-270.

The junior author knows of one informal experiment in which a *third-grade* teacher, after teaching EA, had her pupils change to D, the reverse of the proposal here advanced. The transition was effected by her pupils with no serious loss in efficiency. It is to be supposed that children in later grades could make the change with even less disturbance in skill, and as easily from D to EA as from EA to D.

## BIBLIOGRAPHY

The references which deal directly with the major problem of this investigation are listed at the end of Chapter I, beginning on page 23. In addition to these twenty-nine references, the following have been cited in the monograph:

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## Appendices

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## APPENDIX A

### NOTE ON THE USE OF THE SHEN FORMULA

The Critical Ratios by which differences in means in evaluative measures were calculated were obtained through use of Shen's formula. In the case of the Critical Ratio for rate on Part I of the End Test (Table 14), this method may be illustrated for DR-DM:

$$t = \frac{x_{DR} - x_{DM}}{\sqrt{\frac{\sum(x_{DR} - \bar{x}_{DR})^2 + \sum(x_{DM} - \bar{x}_{DM})^2}{N_{DR} + N_{DM} - 6}} (1 - r_{a.1234}^2) \left( \frac{1}{N_{DR}} + \frac{1}{N_{DM}} \right)}$$

In this formula:

$t$  = Critical Ratio

$\bar{x}$  = mean

$X$  = deviations from mean

$N$  = frequency

6 = the number of variables used in matching (here 4), plus 2

$r_{a.1234}$  = correlation between rate on Part I of the End Test with the matching factors (MA, IQ, rate on Pretest, accuracy on Pretest) combined.

The term  $r_{a.1234}$  is interpreted as follows. The Arabic numerals, 1 through 4, represent the four matching factors utilized for equating the groups of experimental subjects. They are: Mental Age (1), Intelligence Quotient (2), rate on Pretest (3), and accuracy on Pretest (4). The letters  $a$  through  $f$  refer to the final measures as they were correlated with the matching factors. By way of illustration, attention is called to the first of the multiple correlation coefficients listed below. The phrase, Rate on End Test, Part I ( $a$ ):  $r_{a.1234} = 0.47$ , means that the coefficient of multiple correlation between Rate on Part I of the End Test for Center A and the four matching factors combined amounts to 0.47. This coefficient was used in Shen's formula for all Center A Critical Ratios in which Rate on Part I of the End Test was involved. In this connection the same coefficient was employed in the formula for comparisons based upon the whole test, borrowing examples, and nonborrowing examples; separate coefficients were not computed for borrowing and nonborrowing parts of any of the end measures.

For each center six coefficients of multiple correlation were calculated for use in Shen's formula. They are listed below.

|                                                          | Center<br>A | Center<br>B | Center<br>C |
|----------------------------------------------------------|-------------|-------------|-------------|
| Rate on End Test, Part I (a) : $r_{a.1234}$              | 0.47        | 0.34        | 0.50        |
| Accuracy on End Test, Part I (b) : $r_{b.1234}$          | 0.53        | 0.60        | 0.61        |
| Rate on Transfer Test (c) : $r_{c.1234}$                 | 0.35        | 0.42        | 0.42        |
| Accuracy on Transfer Test (d) : $r_{d.1234}$             | 0.38        | 0.50        | 0.43        |
| Smoothness of Performance, Interviews (e) : $r_{e.1234}$ | 0.48        | 0.28        | 0.36        |
| Degree of Understanding, Interviews (f) : $r_{f.1234}$   | 0.41        | 0.28        | 0.26        |

For each center six coefficients of multiple correlation are listed, one coefficient for each of the six experimental factors ( $a - f$ ). Actually, of course, there are twenty-four such coefficients, there being six for each of the four experimental sections in each center. These twenty-four coefficients of multiple correlation were, however, not calculated. Instead, their total number was held to six, as listed above, by using a shortcut involving averages.

To illustrate the method in the case of the Center A coefficient of multiple correlation,  $r_{a.1234}$ : the simple coefficients of correlation between MA and rate on the End Test, Part I, were computed separately for the four experimental sections. These coefficients (DR, 0.43; DM, 0.09; EAR, 0.18; EAM, 0.24) were then averaged, yielding 0.24. The same procedure was used in the case of the matching factors. (The averaging is justified by the fact that the experimental sections had very nearly the same populations.) The average coefficient was then used in obtaining the coefficient of multiple correlation for rate on End Test, Part I, for Center A as a whole.

True, by this method differences introduced by the four types of experimental teaching are in part canceled; but, as a means of lessening labor without introducing too much error, it seemed to be adequate for its purpose.

TABLE 28  
CENTER C—COMPARABILITY OF SECTIONS WITH RESPECT TO IQ

| Section          | $M$   | $\sigma_{dist.}$ | $\sigma M$ |
|------------------|-------|------------------|------------|
| DR (N=125).....  | 97.58 | 11.36            | 1.02       |
| DM (N=123).....  | 95.69 | 10.91            | 0.98       |
| EAR (N=122)..... | 96.11 | 9.51             | 0.86       |
| EAM (N=107)..... | 95.41 | 10.75            | 1.04       |

CRITICAL RATIOS:

DR — DM 1.34  
DR — EAR 1.11  
DR — EAM 2.05

DM — EAM 0.20  
EAR — DM 0.25  
EAR — EAM 0.52

TABLE 29  
CENTER C—COMPARABILITY OF SECTIONS WITH RESPECT TO MA

| <i>Section</i>   | <i>M</i> | $\sigma_{dist.}$ | $\sigma M$ |
|------------------|----------|------------------|------------|
| DR (N=125).....  | 104.03   | 7.77             | 0.70       |
| DM (N=123).....  | 100.71   | 8.51             | 0.77       |
| EAR (N=122)..... | 102.41   | 6.98             | 0.63       |
| EAM (N=107)..... | 100.28   | 8.20             | 0.79       |

## CRITICAL RATIOS:

DR - DM 3.19  
DR - EAR 1.72  
DR - EAM 3.57

DM - EAM 0.39  
EAR - DM 1.70  
EAR - EAM 2.11

TABLE 30  
CENTER C—COMPARABILITY OF SECTIONS WITH RESPECT TO CA

| <i>Section</i>   | <i>M</i> | $\sigma_{dist.}$ | $\sigma M$ |
|------------------|----------|------------------|------------|
| DR (N=125).....  | 107.80   | 10.54            | 0.94       |
| DM (N=123).....  | 106.29   | 7.77             | 0.70       |
| EAR (N=122)..... | 107.32   | 8.68             | 0.79       |
| EAM (N=109)..... | 106.19   | 8.12             | 0.78       |

## CRITICAL RATIOS:

DR - DM 1.29  
DR - EAR 0.39  
DR - EAM 1.32

DM - EAM 0.10  
EAR - DM 0.98  
EAR - EAM 1.02

TABLE 31  
CENTER C—COMPARABILITY OF SECTIONS WITH RESPECT TO  
COMPUTATIONAL ACCURACY

| <i>Section</i>   | <i>M</i> | $\sigma_{dist.}$ | $\sigma M$ |
|------------------|----------|------------------|------------|
| DR (N=125).....  | 19.68    | 6.45             | 0.58       |
| DM (N=123).....  | 18.55    | 6.30             | 0.57       |
| EAR (N=122)..... | 19.91    | 6.90             | 0.62       |
| EAM (N=107)..... | 19.11    | 6.38             | 0.62       |

## CRITICAL RATIOS:

DR - DM 1.38  
DR - EAM 0.67  
EAR - DR 0.27

EAR - DM 1.62  
EAR - EAM 0.92  
EAM - DM 0.67

TABLE 32  
CENTER C—COMPARABILITY OF SECTIONS WITH RESPECT TO RATE  
OF COMPUTATION

| <i>Section</i>   | <i>M*</i> | $\sigma_{dist.}$ | $\sigma M$ |
|------------------|-----------|------------------|------------|
| DR (N=125).....  | 33.86     | 9.74             | 0.87       |
| DM (N=123).....  | 29.39     | 7.66             | 0.69       |
| EAR (N=122)..... | 32.38     | 10.17            | 0.92       |
| EAM (N=107)..... | 33.11     | 12.02            | 1.16       |

\*M equals number of examples attempted in 15 minutes.

CRITICAL RATIOS:

|               |                |
|---------------|----------------|
| DR — DM 4.03  | EAR — DM 2.60  |
| DR — EAR 1.17 | EAM — DM 2.76  |
| DR — EAM 0.52 | EAM — EAR 0.49 |

TABLE 34  
CENTER C—END TEST, PART I: RELATIVE ACCURACY, WHOLE TEST  
(18 EXAMPLES)

| <i>Section</i>   | <i>M</i> | $\sigma_{dist.}$ | $\sigma M$ |
|------------------|----------|------------------|------------|
| DR (N=118).....  | 15.59    | 4.41             | 0.41       |
| DM (N=114).....  | 12.24    | 4.68             | 0.44       |
| EAR (N=112)..... | 14.59    | 4.46             | 0.42       |
| EAM (N=102)..... | 11.78    | 5.60             | 0.55       |

CRITICAL RATIOS:

|                |                 |
|----------------|-----------------|
| DR — DM 13.40  | DM — EAM 1.53   |
| DR — EAM 13.61 | EAR — DM 9.40   |
| DR — EAR 4.35  | EAR — EAM 10.04 |

TABLE 35  
CENTER C—END TEST, PART I: RELATIVE ACCURACY, BORROWING  
EXAMPLES (14 EXAMPLES)

| <i>Section</i>   | <i>M</i> | $\sigma_{dist.}$ | $\sigma M$ |
|------------------|----------|------------------|------------|
| DR (N=118).....  | 12.02    | 3.59             | 0.33       |
| DM (N=113).....  | 10.69    | 4.09             | 0.39       |
| EAR (N=112)..... | 11.43    | 3.69             | 0.35       |
| EAM (N=102)..... | 9.39     | 4.92             | 0.49       |

CRITICAL RATIOS:

|                |                |
|----------------|----------------|
| DR — DM 6.33   | DM — EAM 5.00  |
| DR — EAR 2.95  | EAR — DM 3.36  |
| DR — EAM 10.96 | EAR — EAM 8.16 |

TABLE 36  
CENTER C—END TEST, PART I: ACCURACY ON NONBORROWING  
EXAMPLES (4 EXAMPLES)

| Section  | Per Cents of Subjects Making Scores |    |    |    |    |
|----------|-------------------------------------|----|----|----|----|
|          | 4                                   | 3  | 2  | 1  | 0  |
| DR.....  | 58                                  | 30 | 8  | 2  | 2  |
| DM.....  | 43                                  | 22 | 15 | 10 | 10 |
| EAR..... | 46                                  | 30 | 14 | 5  | 5  |
| EAM..... | 30                                  | 20 | 19 | 12 | 19 |

TABLE 37  
CENTER C—RETENTION TEST: RELATIVE ACCURACY, WHOLE TEST  
(42 EXAMPLES)

| Section          | $M^*$ | $\sigma_{dist.}$ | $\sigma_M$ |
|------------------|-------|------------------|------------|
| DR (N=107).....  | 33.61 | 8.29             | 0.80       |
| DM (N=106).....  | 30.49 | 10.70            | 1.04       |
| EAR (N=103)..... | 31.98 | 9.23             | 0.91       |
| EAM (N=94).....  | 29.70 | 11.46            | 1.18       |

\*The means in this table do not agree exactly with the corresponding means of submeans in Tables 38 and 39. For the explanation, see p. 53 n. 3.

CRITICAL RATIOS:

|               |                |
|---------------|----------------|
| DR — DM 5.67  | DM — EAM 1.15  |
| DR — EAR 3.08 | EAR — DM 2.48  |
| DR — EAM 6.31 | EAR — EAM 3.56 |

TABLE 38  
CENTER C—RETENTION TEST: RELATIVE ACCURACY, BORROWING  
EXAMPLES (32 EXAMPLES)

| Section          | $M$   | $\sigma_{dist.}$ | $\sigma_M$ |
|------------------|-------|------------------|------------|
| DR (N=107).....  | 24.75 | 6.93             | 0.67       |
| DM (N=106).....  | 22.85 | 9.10             | 0.88       |
| EAR (N=103)..... | 23.98 | 8.25             | 0.81       |
| EAM (N=94).....  | 22.09 | 9.90             | 1.02       |

CRITICAL RATIOS:

|               |                |
|---------------|----------------|
| DR — DM 4.04  | DM — EAM 1.29  |
| DR — EAR 1.67 | EAR — DM 2.17  |
| DR — EAM 5.02 | EAR — EAM 3.38 |

TABLE 39  
CENTER C—RETENTION TEST: RELATIVE ACCURACY, NONBORROWING  
EXAMPLES (10 EXAMPLES)

| Section          | <i>M</i> | $\sigma_{dist.}$ | $\sigma M$ |
|------------------|----------|------------------|------------|
| DR (N=108).....  | 8.52     | 1.52             | 0.15       |
| DM (N=106).....  | 7.08     | 2.54             | 0.25       |
| EAR (N=103)..... | 7.48     | 2.18             | 0.22       |
| EAM (N=94).....  | 7.17     | 2.43             | 0.25       |

## CRITICAL RATIOS:

DR — DM 11.08  
DR — EAR 8.00  
DR — EAM 9.64

EAR — DM 2.50  
EAR — EAM 1.82  
EAM — DM 0.53

TABLE 42  
CENTER C—END TEST, PART I: RELATIVE RATE OF WORK

| Section          | <i>M</i> | $\sigma_{dist.}$ | $\sigma M$ |
|------------------|----------|------------------|------------|
| DR (N=118).....  | 16.03    | 3.12             | 0.29       |
| DM (N=114).....  | 16.44    | 3.06             | 0.28       |
| EAR (N=112)..... | 15.62    | 3.76             | 0.36       |
| EAM (N=102)..... | 15.31    | 4.84             | 0.48       |

## CRITICAL RATIOS:

DR — EAR 1.86  
DR — EAM 3.13  
DM — DR 1.95

DM — EAR 3.94  
DM — EAM 5.14  
EAR — EAM 1.29

TABLE 43  
CENTER C—RETENTION TEST: RELATIVE RATE OF WORK

| Section          | <i>M</i> | $\sigma_{dist.}$ | $\sigma M$ |
|------------------|----------|------------------|------------|
| DR (N=107).....  | 29.51    | 8.95             | 0.87       |
| DM (N=106).....  | 24.26    | 8.10             | 0.79       |
| EAR (N=102)..... | 30.25    | 8.82             | 0.87       |
| EAM (N=94).....  | 26.47    | 9.13             | 0.94       |

## CRITICAL RATIOS:

DR — DM 10.29  
DR — EAM 5.24  
EAR — DR 1.30

EAR — DM 11.30  
EAR — EAM 6.30  
EAM — DM 4.02

TABLE 44

CENTER C—INTERVIEW I: SMOOTHNESS OF PERFORMANCE, BORROWING  
EXAMPLES (3 EXAMPLES)

| Section          | Frequency of Scores |     |    |    | <i>M</i> | $\sigma_{dist.}$ | $\sigma_M$ |
|------------------|---------------------|-----|----|----|----------|------------------|------------|
|                  | 1                   | 2   | 3  | 4  |          |                  |            |
| DR (N=113).....  | 68                  | 244 | 18 | 9  | 1.91     | 0.59             | 0.03       |
| DM (N=104).....  | 204                 | 73  | 25 | 10 | 1.49     | 0.78             | 0.04       |
| EAR (N=109)..... | 92                  | 205 | 29 | 1  | 1.81     | 0.59             | 0.03       |
| EAM (N=98).....  | 214                 | 46  | 19 | 15 | 1.44     | 0.83             | 0.05       |

## CRITICAL RATIOS:

DM — DR 7.00  
DM — EAR 5.33  
EAR — DR 2.50

EAM — DR 7.83  
EAM — DM 0.71  
EAM — EAR 6.17

TABLE 45

CENTER C—INTERVIEW II: SMOOTHNESS OF PERFORMANCE, BORROWING  
EXAMPLES (3 EXAMPLES)

| Section          | Frequency of Scores |    |    |    | <i>M</i> | $\sigma_{dist.}$ | $\sigma_M$ |
|------------------|---------------------|----|----|----|----------|------------------|------------|
|                  | 1                   | 2  | 3  | 4  |          |                  |            |
| DR (N=112).....  | 281                 | 35 | 14 | 6  | 1.24     | 0.61             | 0.03       |
| DM (N=100).....  | 224                 | 47 | 19 | 10 | 1.38     | 0.75             | 0.04       |
| EAR (N=105)..... | 235                 | 52 | 20 | 8  | 1.37     | 0.72             | 0.04       |
| EAM (N=86).....  | 198                 | 41 | 13 | 6  | 1.33     | 0.68             | 0.04       |

## CRITICAL RATIOS:

DR — DM 2.80  
DR — EAR 2.60  
DR — EAM 1.50

EAR — DM 0.17  
EAM — DM 0.83  
EAM — EAR 0.67

TABLE 47\*

CENTER C—INTERVIEW I: DEGREE OF UNDERSTANDING, BORROWING  
EXAMPLES (3 EXAMPLES)

| Section          | Frequency of Scores |    |     |     |    | <i>M</i> | $\sigma_{dist.}$ | $\sigma_M$ |
|------------------|---------------------|----|-----|-----|----|----------|------------------|------------|
|                  | 1                   | 2  | 3   | 4   | 5  |          |                  |            |
| DR (N=113).....  | 152                 | 70 | 56  | 33  | 5  | 1.95     | 1.10             | 0.06       |
| DM (N=104).....  | 11                  | 8  | 56  | 197 | 12 | 3.67     | 0.77             | 0.05       |
| EAR (N=109)..... | 44                  | 64 | 170 | 27  | 0  | 2.59     | 0.84             | 0.05       |
| EAM (N=98).....  | 9                   | 5  | 46  | 138 | 43 | 3.83     | 0.87             | 0.06       |

\*Table 47 is comparable to Table 45 for smoothness of performance.

## CRITICAL RATIOS:

DR — DM 21.50  
DR — EAR 8.00  
DR — EAM 20.89

DM — EAM 2.29  
EAR — DM 13.50  
EAR — EAM 15.50

TABLE 48

CENTER C—INTERVIEW II: DEGREE OF UNDERSTANDING, BORROWING  
EXAMPLES (3 EXAMPLES)

| Section          | Frequency of Scores |    |    |     |    | <i>M</i> | $\sigma_{dist.}$ | $\sigma M$ |
|------------------|---------------------|----|----|-----|----|----------|------------------|------------|
|                  | 1                   | 2  | 3  | 4   | 5  |          |                  |            |
| DR (N=112).....  | 161                 | 47 | 32 | 78  | 3  | 2.11     | 1.28             | 0.07       |
| DM (N=100).....  | 18                  | 7  | 55 | 197 | 14 | 3.63     | 0.87             | 0.05       |
| EAR (N=105)..... | 65                  | 91 | 85 | 46  | 6  | 2.44     | 1.06             | 0.06       |
| EAM (N=86).....  | 0                   | 19 | 41 | 150 | 33 | 3.81     | 0.76             | 0.05       |

CRITICAL RATIOS:

DR — DM 16.89  
DR — EAR 3.30  
DR — EAM 18.89

DM — EAM 2.57  
EAR — DM 14.88  
EAR — EAM 17.13

TABLE 50

CENTER C—TRANSFER TEST: ACCURACY, WHOLE TEST  
(18 EXAMPLES)

| Section          | <i>M</i> | $\sigma_{dist.}$ | $\sigma M$ |
|------------------|----------|------------------|------------|
| DR (N=118).....  | 9.71     | 5.12             | 0.47       |
| DM (N=114).....  | 7.14     | 4.98             | 0.47       |
| EAR (N=112)..... | 7.09     | 4.83             | 0.46       |
| EAM (N=102)..... | 4.82     | 3.37             | 0.33       |

CRITICAL RATIOS:

DR — DM 8.29  
DR — EAR 8.73  
DR — EAM 15.77

DM — EAM 7.48  
DM — EAR 0.17  
EAR — EAM 7.57

TABLE 51

CENTER C—TRANSFER TEST: ACCURACY, BORROWING EXAMPLES  
(15 EXAMPLES)

| Section          | <i>M</i> | $\sigma_{dist.}$ | $\sigma M$ |
|------------------|----------|------------------|------------|
| DR (N=118).....  | 6.83     | 4.75             | 0.44       |
| DM (N=113).....  | 4.89     | 4.79             | 0.45       |
| EAR (N=112)..... | 4.84     | 3.80             | 0.36       |
| EAM (N=102)..... | 2.76     | 4.12             | 0.41       |

CRITICAL RATIOS:

DR — DM 6.47  
DR — EAR 7.37  
DR — EAM 14.03

DM — EAR 0.19  
DM — EAM 7.35  
EAR — EAM 8.00

TABLE 52  
CENTER C—TRANSFER TEST: ACCURACY, NONBORROWING  
EXAMPLES (3 EXAMPLES)

| Section  | Per Cents of Subjects Making Scores |    |    |    |
|----------|-------------------------------------|----|----|----|
|          | 3                                   | 2  | 1  | 0  |
| DR.....  | 75                                  | 14 | 8  | 3  |
| DM.....  | 47                                  | 32 | 11 | 10 |
| EAR..... | 50                                  | 30 | 7  | 13 |
| EAM..... | 39                                  | 23 | 17 | 21 |

TABLE 53  
CENTER C—TRANSFER TEST: RELATIVE RATE OF WORK

| Section          | <i>M</i> | $\sigma_{dist.}$ | $\sigma M$ |
|------------------|----------|------------------|------------|
| DR (N=118).....  | 15.56    | 3.10             | 0.29       |
| DM (N=114).....  | 16.53    | 2.52             | 0.24       |
| EAR (N=112)..... | 15.14    | 4.30             | 0.41       |
| EAM (N=102)..... | 15.35    | 3.27             | 0.32       |

## CRITICAL RATIOS:\*

DR — EAM 0.75

DR — EAR 1.62

DM — DR 4.22

DM — EAR 5.79

DM — EAM 4.54

EAM — EAR 0.72

\*The coefficient of multiple correlation used in calculating Critical Ratios for this table was the same as that used for rate of work on Part I of the End Test (Table 42).

TABLE 56  
CENTER B—COMPARABILITY OF SECTIONS WITH RESPECT TO IQ

| Section         | <i>M</i> | $\sigma_{dist.}$ | $\sigma M$ |
|-----------------|----------|------------------|------------|
| DR (N=72).....  | 96.95    | 11.75            | 1.38       |
| DM (N=73).....  | 96.00    | 7.50             | 0.88       |
| EAR (N=71)..... | 97.29    | 10.10            | 1.20       |
| EAM (N=73)..... | 100.04   | 8.30             | 0.97       |

## CRITICAL RATIOS:

DR — DM 0.58

EAR — DM 0.87

EAM — DM 3.08

EAR — DR 0.19

EAM — DR 1.83

EAM — EAR 1.80

TABLE 57  
CENTER B—COMPARABILITY OF SECTIONS WITH RESPECT TO MA

| <i>Section</i>  | <i>M</i> | $\sigma_{dist.}$ | $\sigma M$ |
|-----------------|----------|------------------|------------|
| DR (N=72).....  | 103.34   | 6.95             | 0.81       |
| DM (N=73).....  | 100.65   | 6.70             | 0.78       |
| EAR (N=71)..... | 103.00   | 7.55             | 0.90       |
| EAM (N=73)..... | 103.05   | 8.05             | 0.94       |

## CRITICAL RATIOS:

DR — DM 2.40  
DR — EAR 0.28  
DR — EAM 0.23

EAR — DM 1.97  
EAM — DM 1.97  
EAM — EAR 0.04

TABLE 58  
CENTER B—COMPARABILITY OF SECTIONS WITH RESPECT TO CA

| <i>Section</i>  | <i>M</i> | $\sigma_{dist.}$ | $\sigma M$ |
|-----------------|----------|------------------|------------|
| DR (N=72).....  | 108.60   | 11.15            | 1.31       |
| DM (N=73).....  | 105.72   | 5.99             | 0.70       |
| EAR (N=71)..... | 106.31   | 7.29             | 0.87       |
| EAM (N=73)..... | 103.46   | 5.01             | 0.59       |

## CRITICAL RATIOS:

DR — DM 1.93  
DR — EAR 1.46  
DR — EAM 3.57

DM — EAM 2.46  
EAR — DM 0.53  
EAR — EAM 2.71

TABLE 59  
CENTER B—COMPARABILITY OF SECTIONS WITH RESPECT TO  
COMPUTATIONAL ACCURACY

| <i>Section</i>  | <i>M</i> | $\sigma_{dist.}$ | $\sigma M$ |
|-----------------|----------|------------------|------------|
| DR (N=72).....  | 20.66    | 7.11             | 0.83       |
| DM (N=73).....  | 22.57    | 6.65             | 0.78       |
| EAR (N=71)..... | 23.11    | 7.08             | 0.84       |
| EAM (N=73)..... | 19.75    | 5.27             | 0.61       |

## CRITICAL RATIOS:

DR — EAM 0.89  
DM — EAM 2.86  
DM — DR 1.17

EAR — DR 2.08  
EAR — DM 0.47  
EAR — EAM 3.24

TABLE 60  
CENTER B—COMPARABILITY OF SECTIONS WITH RESPECT TO  
RATE OF COMPUTATION

| Section         | $M^*$ | $\sigma_{dist.}$ | $\sigma_M$ |
|-----------------|-------|------------------|------------|
| DR (N=72).....  | 32.15 | 8.60             | 1.01       |
| DM (N=73).....  | 33.46 | 9.10             | 1.07       |
| EAR (N=71)..... | 36.16 | 7.95             | 0.94       |
| EAM (N=73)..... | 29.56 | 9.00             | 1.05       |

\*M equals number of examples attempted in 15 minutes.

CRITICAL RATIOS:

|          |      |           |      |
|----------|------|-----------|------|
| DR - EAM | 1.77 | EAR - DR  | 2.91 |
| DM - EAM | 2.60 | EAR - DM  | 1.90 |
| DM - DR  | 0.89 | EAR - EAM | 4.68 |

TABLE 61  
CENTER B—END TEST, PART I: RELATIVE ACCURACY, WHOLE  
TEST (18 EXAMPLES)

| Section         | $M$   | $\sigma_{dist.}$ | $\sigma_M$ |
|-----------------|-------|------------------|------------|
| DR (N=72).....  | 15.39 | 4.20             | 0.50       |
| DM (N=73).....  | 13.70 | 4.34             | 0.51       |
| EAR (N=71)..... | 13.55 | 5.08             | 0.60       |
| EAM (N=73)..... | 14.71 | 4.48             | 0.53       |

CRITICAL RATIOS:

|          |      |           |      |
|----------|------|-----------|------|
| DR - DM  | 5.63 | DM - EAR  | 0.43 |
| DR - EAR | 5.41 | EAM - DM  | 3.26 |
| DR - EAM | 2.19 | EAM - EAR | 3.31 |

TABLE 62  
CENTER B—END TEST, PART I: RELATIVE ACCURACY, BORROWING  
EXAMPLES (14 EXAMPLES)

| Section         | $M$   | $\sigma_{dist.}$ | $\sigma_M$ |
|-----------------|-------|------------------|------------|
| DR (N=72).....  | 12.08 | 3.80             | 0.45       |
| DM (N=73).....  | 11.03 | 3.75             | 0.44       |
| EAR (N=71)..... | 10.33 | 4.65             | 0.56       |
| EAM (N=73)..... | 11.82 | 3.90             | 0.47       |

CRITICAL RATIOS:

|          |      |           |      |
|----------|------|-----------|------|
| DR - DM  | 3.50 | DM - EAR  | 2.12 |
| DR - EAR | 5.47 | EAM - DM  | 2.55 |
| DR - EAM | 0.87 | EAM - EAR | 4.48 |

TABLE 63  
CENTER B—END TEST, PART I: ACCURACY ON NONBORROWING  
EXAMPLES (4 EXAMPLES)

| Section  | Per Cents of Subjects Making Scores |    |    |    |   |
|----------|-------------------------------------|----|----|----|---|
|          | 4                                   | 3  | 2  | 1  | 0 |
| DR.....  | 67                                  | 18 | 12 | 0  | 3 |
| DM.....  | 40                                  | 31 | 11 | 11 | 7 |
| EAR..... | 49                                  | 37 | 7  | 6  | 1 |
| EAM..... | 38                                  | 30 | 14 | 10 | 8 |

TABLE 64  
CENTER B—RETENTION TEST: RELATIVE ACCURACY, WHOLE TEST  
(42 EXAMPLES)

| Section         | <i>M</i> | $\sigma_{dist.}$ | $\sigma M$ |
|-----------------|----------|------------------|------------|
| DR (N=63).....  | 37.44    | 6.32             | 0.80       |
| DM (N=66).....  | 30.21    | 11.79            | 1.45       |
| EAR (N=64)..... | 31.63    | 9.84             | 1.23       |
| EAM (N=65)..... | 32.11    | 9.43             | 1.17       |

## CRITICAL RATIOS:

|          |      |           |      |
|----------|------|-----------|------|
| DR — DM  | 9.77 | EAR — DM  | 1.71 |
| DR — EAR | 9.37 | EAM — DM  | 2.35 |
| DR — EAM | 8.74 | EAM — EAR | 0.66 |

TABLE 65  
CENTER B—RETENTION TEST: RELATIVE ACCURACY, BORROWING  
EXAMPLES (32 EXAMPLES)

| Section         | <i>M</i> | $\sigma_{dist.}$ | $\sigma M$ |
|-----------------|----------|------------------|------------|
| DR (N=63).....  | 28.49    | 5.23             | 0.66       |
| DM (N=66).....  | 22.55    | 10.11            | 1.25       |
| EAR (N=64)..... | 23.62    | 8.66             | 1.08       |
| EAM (N=66)..... | 24.70    | 9.34             | 1.03       |

## CRITICAL RATIOS:

|          |      |           |      |
|----------|------|-----------|------|
| DR — DM  | 9.28 | EAR — DM  | 0.10 |
| DR — EAR | 8.70 | EAM — DM  | 3.07 |
| DR — EAM | 7.02 | EAM — EAR | 1.66 |

TABLE 66

CENTER B—RETENTION TEST: RELATIVE ACCURACY, NONBORROWING  
EXAMPLES (10 EXAMPLES)

| <i>Section</i>  | <i>M</i> | $\sigma_{dist.}$ | $\sigma M$ |
|-----------------|----------|------------------|------------|
| DR (N=63).....  | 9.54     | 1.68             | 0.21       |
| DM (N=66).....  | 8.12     | 2.39             | 0.29       |
| EAR (N=64)..... | 8.44     | 1.92             | 0.24       |
| EAM (N=65)..... | 8.05     | 2.28             | 0.28       |

## CRITICAL RATIOS:

DR — DM 8.35  
DR — EAR 7.86  
DR — EAM 8.77

DM — EAM 0.37  
EAR — DM 1.88  
EAR — EAM 2.29

TABLE 67

CENTER B—PER CENTS OF SUBJECTS IN EXPERIMENTAL SECTIONS MAKING  
INDICATED SCORES ON NONBORROWING EXAMPLES

| <i>Test</i>                   | <i>Per Cents of Subjects, by Sections</i> |           |            |            |
|-------------------------------|-------------------------------------------|-----------|------------|------------|
|                               | <i>DR</i>                                 | <i>DM</i> | <i>EAR</i> | <i>EAM</i> |
| End Test, Part I (4 examples) |                                           |           |            |            |
| Scores of 3 or 4.....         | 85                                        | 71        | 86         | 68         |
| Scores of 1 or 0.....         | 3                                         | 18        | 7          | 18         |
| Retention Test* (10 examples) |                                           |           |            |            |
| Scores of 8, 9, 10.....       | 89                                        | 61        | 69         | 55         |
| Scores of 3, 2, 1, 0.....     | 2                                         | 6         | 5          | 6          |

\*Basic tables not reported in the monograph.

TABLE 68

CENTER B—END TEST, PART I: RELATIVE RATE OF WORK

| <i>Section</i>               | <i>M</i> | $\sigma_{dist.}$ | $\sigma M$ |
|------------------------------|----------|------------------|------------|
| DR <sup>†</sup> (N=72).....  | 16.25    | 2.54             | 0.30       |
| DM <sup>‡</sup> (N=73).....  | 16.27    | 3.14             | 0.37       |
| EAR (N=71).....              | 15.58    | 3.04             | 0.36       |
| EAM <sub>s</sub> (N=73)..... | 16.85    | 2.38             | 0.28       |

## CRITICAL RATIOS:

DR — EAR 2.68  
DM — DR 0.07  
DM — EAR 2.56

EAM — DR 2.61  
EAM — DM 2.32  
EAM — EAR 5.52

TABLE 69

CENTER B—RETENTION TEST: RELATIVE RATE OF WORK

| <i>Section</i>  | <i>M</i> | $\sigma_{dist.}$ | $\sigma M$ |
|-----------------|----------|------------------|------------|
| DR (N=63).....  | 30.87    | 8.37             | 1.06       |
| DM (N=66).....  | 27.33    | 10.51            | 1.29       |
| EAR (N=64)..... | 27.69    | 7.52             | 0.94       |
| EAM (N=65)..... | 29.15    | 7.39             | 0.92       |

CRITICAL RATIOS:

DR — DM 4.37

EAR — DM 0.46

DR — EAR 4.54

EAM — DM 2.43

DR — EAM 2.57

EAM — EAR 2.32

TABLE 70

CENTER B—INTERVIEW I: SMOOTHNESS OF PERFORMANCE, BORROWING  
EXAMPLES (3 EXAMPLES)

| <i>Section</i>  | <i>Frequency of Scores</i> |          |          |          | <i>M</i> | $\sigma_{dist.}$ | $\sigma M$ |
|-----------------|----------------------------|----------|----------|----------|----------|------------------|------------|
|                 | <i>1</i>                   | <i>2</i> | <i>3</i> | <i>4</i> |          |                  |            |
| DR (N=67).....  | 71                         | 111      | 19       | 6        | 1.51     | 0.72             | 0.05       |
| DM (N=66).....  | 129                        | 15       | 43       | 11       | 1.68     | 0.99             | 0.07       |
| EAR (N=67)..... | 22                         | 173      | 10       | 5        | 1.99     | 0.50             | 0.03       |
| EAM (N=69)..... | 163                        | 5        | 22       | 17       | 1.48     | 0.98             | 0.07       |

CRITICAL RATIOS:

DR — DM 1.89

EAM — DR 0.33

DR — EAR 24.00

EAM — DM 1.82

DM — EAR 3.44

EAM — EAR 6.38

TABLE 71

CENTER B—INTERVIEW I: DEGREE OF UNDERSTANDING, BORROWING  
EXAMPLES (3 EXAMPLES)

| <i>Section</i>  | <i>Frequency of Scores</i> |          |          |          |          | <i>M</i> | $\sigma_{dist.}$ | $\sigma M$ |
|-----------------|----------------------------|----------|----------|----------|----------|----------|------------------|------------|
|                 | <i>1</i>                   | <i>2</i> | <i>3</i> | <i>4</i> | <i>5</i> |          |                  |            |
| DR (N=68).....  | 93                         | 9        | 1        | 92       | 9        | 2.58     | 1.52             | 0.11       |
| DM (N=69).....  | 0                          | 0        | 6        | 173      | 31       | 4.12     | 0.40             | 0.03       |
| EAR (N=63)..... | 78                         | 21       | 69       | 12       | 9        | 2.22     | 1.19             | 0.09       |
| EAM (N=64)..... | 0                          | 3        | 45       | 151      | 0        | 3.74     | 0.47             | 0.03       |

CRITICAL RATIOS:

DR — DM 14.00

EAR — DM 21.11

DR — EAM 10.55

EAR — EAM 16.89

EAR — DR 2.57

EAM — DM 9.50

TABLE 72  
CENTER B—TRANSFER TEST: ACCURACY, WHOLE TEST  
(18 EXAMPLES)

| <i>Section</i>  | <i>M</i> | <i>σ<sub>dist.</sub></i> | <i>σ<sub>M</sub></i> |
|-----------------|----------|--------------------------|----------------------|
| DR (N=72).....  | 9.44     | 5.24                     | 0.62                 |
| DM (N=73).....  | 7.09     | 5.10                     | 0.60                 |
| EAR (N=71)..... | 6.37     | 4.04                     | 0.48                 |
| EAM (N=73)..... | 7.24     | 4.76                     | 0.56                 |

## CRITICAL RATIOS:

DR — DM 6.53  
DR — EAR 8.30  
DR — EAM 5.95

DM — EAR 1.95  
EAM — DM 0.41  
EAM — EAR 2.56

TABLE 73  
CENTER B—TRANSFER TEST: ACCURACY, BORROWING EXAMPLES  
(15 EXAMPLES)

| <i>Section</i>  | <i>M</i> | <i>σ<sub>dist.</sub></i> | <i>σ<sub>M</sub></i> |
|-----------------|----------|--------------------------|----------------------|
| DR (N=72).....  | 6.81     | 4.90                     | 0.58                 |
| DM (N=73).....  | 5.08     | 4.52                     | 0.53                 |
| EAR (N=71)..... | 3.79     | 3.68                     | 0.44                 |
| EAM (N=73)..... | 5.36     | 4.37                     | 0.51                 |

## CRITICAL RATIOS:

DR — DM 4.81  
DR — EAR 9.15  
DR — EAM 4.26

DM — EAR 3.91  
EAM — DM 0.80  
EAM — EAR 5.06

TABLE 74  
CENTER B—TRANSFER TEST: ACCURACY NONBORROWING EXAMPLES  
(3 EXAMPLES)

| <i>Section</i> | <i>Per Cents of Subjects Making Scores</i> |    |    |    |
|----------------|--------------------------------------------|----|----|----|
|                | 3                                          | 2  | 1  | 0  |
| DR.....        | 81                                         | 14 | 1  | 4  |
| DM.....        | 52                                         | 27 | 8  | 13 |
| EAR.....       | 75                                         | 17 | 5  | 3  |
| EAM.....       | 47                                         | 24 | 19 | 10 |

TABLE 75  
CENTER B—TRANSFER TEST: RELATIVE RATE OF WORK

| <i>Section</i>  | <i>M</i> | <i>°dist.</i> | <i>°M</i> |
|-----------------|----------|---------------|-----------|
| DR (N=72).....  | 16.56    | 2.34          | 0.28      |
| DM (N=73).....  | 15.48    | 3.62          | 0.42      |
| EAR (N=71)..... | 16.17    | 2.62          | 0.31      |
| EAM (N=73)..... | 16.82    | 2.52          | 0.30      |

## CRITICAL RATIOS:

DR — DM 3.72  
DR — EAR 1.70  
EAR — DM 2.30

EAM — DR 1.24  
EAM — DM 4.62  
EAM — EAR 2.83

# APPENDIX B

## TESTS AND INTERVIEWS

### PRETEST

Write just the answers.

Add or subtract. Watch the signs!

Don't hurry so fast that you will make mistakes.

|                                                    |                                                    |                                                     |                                                     |                                                    |                                                     |                                                     |
|----------------------------------------------------|----------------------------------------------------|-----------------------------------------------------|-----------------------------------------------------|----------------------------------------------------|-----------------------------------------------------|-----------------------------------------------------|
| $\begin{array}{r} 98 \\ - 4 \\ \hline \end{array}$ | $\begin{array}{r} 67 \\ - 3 \\ \hline \end{array}$ | $\begin{array}{r} 43 \\ + 21 \\ \hline \end{array}$ | $\begin{array}{r} 81 \\ + 78 \\ \hline \end{array}$ | $\begin{array}{r} 78 \\ - 5 \\ \hline \end{array}$ | $\begin{array}{r} 37 \\ + 54 \\ \hline \end{array}$ | $\begin{array}{r} 92 \\ - 70 \\ \hline \end{array}$ |
|----------------------------------------------------|----------------------------------------------------|-----------------------------------------------------|-----------------------------------------------------|----------------------------------------------------|-----------------------------------------------------|-----------------------------------------------------|

|                                                     |                                                     |                                                       |                                                       |                                                       |                                                     |                                                      |
|-----------------------------------------------------|-----------------------------------------------------|-------------------------------------------------------|-------------------------------------------------------|-------------------------------------------------------|-----------------------------------------------------|------------------------------------------------------|
| $\begin{array}{r} 75 \\ + 69 \\ \hline \end{array}$ | $\begin{array}{r} 63 \\ - 62 \\ \hline \end{array}$ | $\begin{array}{r} 375 \\ + 624 \\ \hline \end{array}$ | $\begin{array}{r} 490 \\ - 360 \\ \hline \end{array}$ | $\begin{array}{r} 385 \\ + 207 \\ \hline \end{array}$ | $\begin{array}{r} 45 \\ - 14 \\ \hline \end{array}$ | $\begin{array}{r} 177 \\ - 56 \\ \hline \end{array}$ |
|-----------------------------------------------------|-----------------------------------------------------|-------------------------------------------------------|-------------------------------------------------------|-------------------------------------------------------|-----------------------------------------------------|------------------------------------------------------|

|                                                      |                                                       |                                                     |                                                      |                                                      |                                                     |                                                      |
|------------------------------------------------------|-------------------------------------------------------|-----------------------------------------------------|------------------------------------------------------|------------------------------------------------------|-----------------------------------------------------|------------------------------------------------------|
| $\begin{array}{r} 396 \\ - 42 \\ \hline \end{array}$ | $\begin{array}{r} 851 \\ + 929 \\ \hline \end{array}$ | $\begin{array}{r} 89 \\ - 68 \\ \hline \end{array}$ | $\begin{array}{r} 885 \\ - 33 \\ \hline \end{array}$ | $\begin{array}{r} 367 \\ - 34 \\ \hline \end{array}$ | $\begin{array}{r} 6 \\ + 1 \\ \hline 1 \end{array}$ | $\begin{array}{r} 655 \\ - 25 \\ \hline \end{array}$ |
|------------------------------------------------------|-------------------------------------------------------|-----------------------------------------------------|------------------------------------------------------|------------------------------------------------------|-----------------------------------------------------|------------------------------------------------------|

|                                                       |                                                       |                                                       |                                                     |                                                       |                                                        |                                                       |
|-------------------------------------------------------|-------------------------------------------------------|-------------------------------------------------------|-----------------------------------------------------|-------------------------------------------------------|--------------------------------------------------------|-------------------------------------------------------|
| $\begin{array}{r} 759 \\ - 715 \\ \hline \end{array}$ | $\begin{array}{r} 11 \\ + 6 \\ \hline 70 \end{array}$ | $\begin{array}{r} 861 \\ - 751 \\ \hline \end{array}$ | $\begin{array}{r} 5 \\ + 1 \\ \hline 5 \end{array}$ | $\begin{array}{r} 673 \\ - 123 \\ \hline \end{array}$ | $\begin{array}{r} 42 \\ + 41 \\ \hline 63 \end{array}$ | $\begin{array}{r} 894 \\ - 210 \\ \hline \end{array}$ |
|-------------------------------------------------------|-------------------------------------------------------|-------------------------------------------------------|-----------------------------------------------------|-------------------------------------------------------|--------------------------------------------------------|-------------------------------------------------------|

|                                                      |                                                      |                                                      |                                                        |                                                      |                                                            |                                                      |
|------------------------------------------------------|------------------------------------------------------|------------------------------------------------------|--------------------------------------------------------|------------------------------------------------------|------------------------------------------------------------|------------------------------------------------------|
| $\begin{array}{r} 127 \\ - 87 \\ \hline \end{array}$ | $\begin{array}{r} 145 \\ - 61 \\ \hline \end{array}$ | $\begin{array}{r} 189 \\ - 95 \\ \hline \end{array}$ | $\begin{array}{r} 53 \\ + 34 \\ \hline 85 \end{array}$ | $\begin{array}{r} 163 \\ - 71 \\ \hline \end{array}$ | $\begin{array}{r} 3 \\ + 7 \\ \hline 8 \\ + 2 \end{array}$ | $\begin{array}{r} 140 \\ - 50 \\ \hline \end{array}$ |
|------------------------------------------------------|------------------------------------------------------|------------------------------------------------------|--------------------------------------------------------|------------------------------------------------------|------------------------------------------------------------|------------------------------------------------------|

|                                                             |                                                      |                                                        |                                                    |                                                      |                                                      |                                                             |
|-------------------------------------------------------------|------------------------------------------------------|--------------------------------------------------------|----------------------------------------------------|------------------------------------------------------|------------------------------------------------------|-------------------------------------------------------------|
| $\begin{array}{r} 2 \\ 2 \\ 2 \\ + 3 \\ \hline \end{array}$ | $\begin{array}{r} 109 \\ - 82 \\ \hline \end{array}$ | $\begin{array}{r} 37 \\ + 5 \\ \hline 439 \end{array}$ | $\begin{array}{r} 44 \\ - 2 \\ \hline \end{array}$ | $\begin{array}{r} 149 \\ - 80 \\ \hline \end{array}$ | $\begin{array}{r} 108 \\ - 30 \\ \hline \end{array}$ | $\begin{array}{r} 7 \\ 8 \\ 6 \\ + 9 \\ \hline \end{array}$ |
|-------------------------------------------------------------|------------------------------------------------------|--------------------------------------------------------|----------------------------------------------------|------------------------------------------------------|------------------------------------------------------|-------------------------------------------------------------|

|                                                                   |                                                      |                                                       |                                                                   |                                                       |                                                    |                                                      |
|-------------------------------------------------------------------|------------------------------------------------------|-------------------------------------------------------|-------------------------------------------------------------------|-------------------------------------------------------|----------------------------------------------------|------------------------------------------------------|
| $\begin{array}{r} \$4.26 \\ 3.32 \\ + 3.00 \\ \hline \end{array}$ | $\begin{array}{r} 124 \\ - 54 \\ \hline \end{array}$ | $\begin{array}{r} 929 \\ - 223 \\ \hline \end{array}$ | $\begin{array}{r} \$8.12 \\ 3.04 \\ + 4.54 \\ \hline \end{array}$ | $\begin{array}{r} 908 \\ - 903 \\ \hline \end{array}$ | $\begin{array}{r} 56 \\ - 4 \\ \hline \end{array}$ | $\begin{array}{r} 177 \\ - 97 \\ \hline \end{array}$ |
|-------------------------------------------------------------------|------------------------------------------------------|-------------------------------------------------------|-------------------------------------------------------------------|-------------------------------------------------------|----------------------------------------------------|------------------------------------------------------|

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A \_\_\_\_\_ S \_\_\_\_\_ To \_\_\_\_\_ Ti \_\_\_\_\_

END TEST  
(INCLUDING TRANSFER TEST)

|                               |                               |                               |                               |                               |                               |                               |
|-------------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| Part I<br>(Process<br>taught) | (1)<br>232<br>— 128<br><hr/>  | (2)<br>83<br>— 45<br><hr/>    | (3)<br>679<br>— 263<br><hr/>  | (4)<br>545<br>— 507<br><hr/>  | (5)<br>483<br>— 57<br><hr/>   | (6)<br>656<br>— 319<br><hr/>  |
|                               | (7)<br>176<br>— 80<br><hr/>   | (8)<br>571<br>— 143<br><hr/>  | (9)<br>853<br>— 623<br><hr/>  | (10)<br>972<br>— 769<br><hr/> | (11)<br>84<br>— 19<br><hr/>   | (12)<br>871<br>— 759<br><hr/> |
|                               | (13)<br>993<br>— 438<br><hr/> | (14)<br>585<br>— 439<br><hr/> | (15)<br>497<br>— 427<br><hr/> | (16)<br>896<br>— 527<br><hr/> | (17)<br>494<br>— 427<br><hr/> | (18)<br>151<br>— 38<br><hr/>  |

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|                                |                               |                               |                               |                               |                               |                               |
|--------------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| Part II<br>(Process<br>taught) | (1)<br>465<br>— 408<br><hr/>  | (2)<br>391<br>— 138<br><hr/>  | (3)<br>798<br>— 364<br><hr/>  | (4)<br>64<br>— 35<br><hr/>    | (5)<br>392<br>— 284<br><hr/>  | (6)<br>582<br>— 473<br><hr/>  |
|                                | (7)<br>179<br>— 99<br><hr/>   | (8)<br>73<br>— 18<br><hr/>    | (9)<br>848<br>— 502<br><hr/>  | (10)<br>996<br>— 417<br><hr/> | (11)<br>451<br>— 322<br><hr/> | (12)<br>883<br>— 26<br><hr/>  |
|                                | (13)<br>584<br>— 247<br><hr/> | (14)<br>782<br>— 159<br><hr/> | (15)<br>648<br>— 516<br><hr/> | (16)<br>171<br>— 24<br><hr/>  | (17)<br>975<br>— 736<br><hr/> | (18)<br>276<br>— 169<br><hr/> |

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|                                |                               |                              |                               |                               |                               |                               |
|--------------------------------|-------------------------------|------------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| Part III<br>(Transfer<br>Test) | (1)<br>539<br>— 242<br><hr/>  | (2)<br>388<br>— 98<br><hr/>  | (3)<br>978<br>— 623<br><hr/>  | (4)<br>307<br>— 65<br><hr/>   | (5)<br>845<br>— 562<br><hr/>  | (6)<br>729<br>— 55<br><hr/>   |
|                                | (7)<br>308<br>— 236<br><hr/>  | (8)<br>648<br>— 395<br><hr/> | (9)<br>579<br>— 464<br><hr/>  | (10)<br>720<br>— 78<br><hr/>  | (11)<br>845<br>— 196<br><hr/> | (12)<br>362<br>— 97<br><hr/>  |
|                                | (13)<br>871<br>— 485<br><hr/> | (14)<br>424<br>— 48<br><hr/> | (15)<br>996<br>— 374<br><hr/> | (16)<br>730<br>— 569<br><hr/> | (17)<br>500<br>— 114<br><hr/> | (18)<br>863<br>— 777<br><hr/> |

## POSTEXPERIMENTAL TEST

|       |       |       |       |       |
|-------|-------|-------|-------|-------|
| 711   | 483   | 698   | 85    | 572   |
| - 408 | - 69  | - 92  | - 27  | - 465 |
| <hr/> | <hr/> | <hr/> | <hr/> | <hr/> |
| 157   | 135   | 793   | 862   | 407   |
| - 148 | - 60  | - 277 | - 213 | - 104 |
| <hr/> | <hr/> | <hr/> | <hr/> | <hr/> |
| 898   | 960   | 384   | 145   | 895   |
| - 253 | - 756 | - 65  | - 96  | - 539 |
| <hr/> | <hr/> | <hr/> | <hr/> | <hr/> |
| 470   | 862   | 367   | 74    | 614   |
| - 413 | - 424 | - 350 | - 17  | - 305 |
| <hr/> | <hr/> | <hr/> | <hr/> | <hr/> |

## RETENTION TEST

|        |        |        |        |        |        |
|--------|--------|--------|--------|--------|--------|
| 214    | 780    | \$6.72 | 793    | 165    | \$2.46 |
| - 209  | - 156  | - .39  | - 276  | - 83   | - .39  |
| <hr/>  | <hr/>  | <hr/>  | <hr/>  | <hr/>  | <hr/>  |
| 154    | \$8.70 | 92     | \$5.95 | 827    | 570    |
| - 48   | - 8.65 | - 55   | - 5.90 | - 629  | - 43   |
| <hr/>  | <hr/>  | <hr/>  | <hr/>  | <hr/>  | <hr/>  |
| \$5.51 | 745    | \$1.78 | 472    | 881    | 79     |
| - 4.16 | - 36   | - .63  | - 223  | - 477  | - 45   |
| <hr/>  | <hr/>  | <hr/>  | <hr/>  | <hr/>  | <hr/>  |
| 851    | \$4.38 | 297    | 86     | \$1.56 | 935    |
| - 208  | - 3.08 | - 118  | - 37   | - .96  | - 718  |
| <hr/>  | <hr/>  | <hr/>  | <hr/>  | <hr/>  | <hr/>  |
| \$8.40 | 683    | 935    | \$7.96 | 670    | 312    |
| - .40  | - 514  | - 627  | - 7.90 | - 438  | - 204  |
| <hr/>  | <hr/>  | <hr/>  | <hr/>  | <hr/>  | <hr/>  |
| 124    | \$9.81 | 590    | 882    | \$6.43 | 94     |
| - 15   | - .74  | - 240  | - 58   | - 5.35 | - 29   |
| <hr/>  | <hr/>  | <hr/>  | <hr/>  | <hr/>  | <hr/>  |
| 531    | 104    | \$6.12 | 371    | 852    | \$4.40 |
| - 305  | - 73   | - 6.07 | - 29   | - 516  | - 4.19 |
| <hr/>  | <hr/>  | <hr/>  | <hr/>  | <hr/>  | <hr/>  |

## INSTRUCTIONS FOR INTERVIEWS

## PURPOSE OF INTERVIEWS

The purposes of the interviews are three in number. We want to know:

- (1) *How* each child subtracts (whether by D or by EA).
- (2) *How well* each child subtracts (the smoothness of his work).
- (3) How thoroughly he *understands* what he does when he borrows.

## GENERAL INSTRUCTIONS

1. The interview technique seems disarmingly simple; but accurate and reliable data are not easily obtained. You must study the directions very carefully, must know the code symbols perfectly, and must be skilled in your questioning. On this account you will be given a series of directed practices, and at times during the experimental interviewing you will be checked by a "supervisor" in order to assure exact conformity to instructions.

2. You must be especially careful to avoid leading questions and reasonable suggestions, for children will immediately adopt any useful cue. Instead, you must proceed *indirectly* to obtain your data, sometimes eliciting them by offering absurd explanations or suggestions.

3. In the interviews we are not concerned with combination errors. Rather, we are concerned with the *process* of subtracting, and weakness on the combinations is significant only when it interferes with smoothness of operation.

4. "Smoothness" or quality of subtracting is not wholly a matter of speed, such that the fastest child is necessarily the smoothest and draws the highest rating, whereas a slower child is rated low in quality. Smoothness means chiefly the absence of jerkiness, of unevenness. When a child subtracts at what seems to be a normal rate *for him*, and does so steadily, he should have credit for high quality of operation.

5. To interview you will need: (a) a sheet of examples to be worked by the children and (b) a record blank. The examples, eight in a row, are mimeographed so that one row can be used with each child (the other rows being kept folded under out of sight). The record blank contains spaces for eleven children, with two sample records to illustrate the method of recording and a list of the code symbols with brief explanations.

6. Make your entries as you go. Do not rely upon your memory to fill in the code symbols after you are through with a subject. You are almost certain to forget some significant fact about the child's procedure.

7. Whenever you are in doubt about classifying any aspect of a child's processes, copy down his exact statements *verbatim*. Then consult your

"supervisor" at the earliest opportunity. This procedure is adopted in order to avoid guessing and to insure agreement for all pupils interviewed.

#### STEPS IN INTERVIEWING

1. Talk informally with the child for a few minutes. (This puts him at his ease and establishes rapport.)

2. Lay the row of eight examples before him, all except the first covered by a card or slip of paper.

3. *Example 1:* Have the child work this example *silently*, putting his work on the paper. Observe (a) the use or nonuse of a crutch and (b) the quality or smoothness of his work. Make no entries on the record blank for this example. (You may not get evidence on the use of a crutch at this time. Step 11 below provides another opportunity.)

4. *Example 2:* Expose this example, and ask the child to work it orally ("out loud") as he enters the figures of his answer. Make no entries on the record blank. If the child does not make his procedure completely clear, copy another example or two and give further training. As soon as you are sure that the child will henceforth give you the information you want, say, "Fine; now I want you to work some more examples in the same way."

5. *Example 3, first test example:* Expose this example, and have the child work it as in Step 4. At this time enter data only for Row A (later for Row B; Step 10 below).

In the first space after A, enter D or EA to tell how the child has worked the example.

In the second space enter C or N (noncrutch method) if you already know it, and follow by the correct digit, 1 to 5, to record the quality of his performance. (See page 184 for the meaning of the code letters and numbers). If you have no information as to C or N, enter only the figure to describe the quality of performance.

6. *Example 4, second test example:* Expose this example, in which no borrowing is involved. Hence, enter N (the crutch cannot be used), and the digit which describes the quality of the performance.

7. *Example 5, third test example:* Proceed as with Example 3, Step 5.

8. *Example 6, fourth test example:* Proceed as with Example 3, Step 5.

9. *Example 7, fifth test example:* Proceed as with Example 4, Step 6.

10. Return now to the three borrowing examples, in order to secure data for Row B. For each of these three examples enter 0 together with the number 1 to 5 to record the degree of understanding revealed. (See page 184 for explanation of the code digits.)

11. If Step 3 above did not disclose use of the crutch, make use of Example 8, which you should alter to show the D-crutch or the EA-crutch. Ask the child whether he ever uses (or has used) this "picture." If the answer is "Yes," enter C in the second spaces in Row A; if "No," enter N in these spaces.

12. Do not stop at this time to fill in the data called for in the last column under "Summary."

13. Fold under the row of examples which the subject has used and go on with the next subject.

*Note:* After the first interviews, your "supervisor" will confer with you and with the other interviewers in your group, in order to agree on the symbols to be used.

#### DIRECTIONS AND QUESTIONS

*For Example 1*, say, "Take your pencil and work the first example for me. Work it just the way you always do when your teacher gives you examples to do at your seat."

*For Example 2*, say, "Work this example out loud, so that I can hear what you are thinking. Work just as if I weren't here, but work out loud. You don't need to tell me why you do things. Just say the words you think when you work the example."

*For Examples 3 through 8*, first time through, say, "Now I want you to work the next five examples in the same way. Say out loud what you think, but don't bother to tell me why you do what you do. Just pretend that I am not here—but talk out loud." (These directions are intended to help you get accurate information on the quality of the process, by forestalling the explanations which you will want later for row B, but which now would complicate matters too much.)

*For Examples 3, 5, and 6*, the second time through, to get data for Row B (illustrated from Example 3), say, "In this example you said that you took 8 from 15. But I don't see 15; I see 5. How did you get 15?"

If the child says that he borrowed or added a 1, ask, "But where did you get the 1?" Then, later, "But if you put the 1 with 5, you would get 6, wouldn't you? and not 15?" *Do not mention* the word *ten* at all, unless the child himself volunteers it.

If the child says that he borrowed or added a ten, make sure that he really understands that it *is* a ten, and that he knows how and where he got it—by borrowing a ten from the tens' figure of the minuend in D; and, in EA, by adding a ten to the ones' figure of the minuend and, at the same time, to the tens' figure of the subtrahend.

Then ask, "And what did you say here?," pointing to the tens' column. Then, "And why did you say that?," leading the child to explain what has happened as the result of borrowing.

*For Example 8*, if you need to use it, alter the example as at the right, depending upon whether the child has shown that he habitually uses D or EA. Then say, "How do you remember that you make all the changes when you subtract? Do you ever help yourself by writing little marks like this?" If the answer is "Yes," then C, rather than D, appears in the second space in Row A for *all* borrowing examples.)

|                                                           |                                                           |
|-----------------------------------------------------------|-----------------------------------------------------------|
| D                                                         | EA                                                        |
| $\begin{array}{r} 4 \overline{) 214} \\ -218 \end{array}$ | $\begin{array}{r} 4 \overline{) 318} \\ -214 \end{array}$ |

## CODE LETTERS AND DIGITS

For borrowing examples (Examples 3, 5, and 6)

Row A: first space, record D or EA to show method second space:

C for crutch, whether written into the example or used mentally.

N if no crutch is used, regardless of the pattern of the process used.

Whether C or N, enter the digit 1, 2, 3, or 4—to describe the quality of the process. (See below.)

Row A:  $C_1$  = Used crutch *mentally*, i.e., did not write it out. Performance even and smooth and prompt (for the given child).

$C_2$  = Crutch *written out*; only difference from  $C_1$ . Same prompt, confident performance as  $C_1$ .

$C_3$  = Crutch written out *or* used mentally, but procedure marked by mental exploration; hesitation as child tries to “piece out” the procedure. Pattern not well fixed.

$C_4$  = Crutch written *or* mental, but procedure erratic and uncertain; much confusion. Inaccurate application of steps in borrowing.

$N_1$  } See descriptions for N  
 $N_2$  } opposite; alter to adapt  
 $N_3$  } to borrowing examples.  
 $N_4$  }

For nonborrowing examples (Examples 4, 7)

N is the only probable entry, because the crutch should not be used. If the child tries to use the crutch anyhow, of course you will enter C.

After N, enter the digit 1, 2, 3, or 4—to describe the quality of the process. (See below.)

$N_1$  = No crutch used here or elsewhere below. Any other process to be designated as N.

$N_1$  means no hesitation; process well habituated.

$N_2$  = In general the process is well habituated, but the performance for *either* tens' or ones' figures is inferior to that for the other. In other words, not all steps performed with equal proficiency.

$N_3$  = Process not well habituated; evidence of trouble with *both* tens' and ones' figures. Hesitancy, as if trying to remember rules.

$N_4$  = Confusion throughout, as in the case of  $C_4$  opposite.

x = If child in subtracting tens or hundreds acts *as if* he has borrowed when he has not done so, enter X after the rest of the entry, as  $N_1x$  or  $N_4x$ .

y = child in subtracting in tens' place forgets he has borrowed.

Row B: How well does the child understand what he does? (Evaluation based upon *whole* performance, for both tens and ones.)

$O_1$  = Understands that the “1” borrowed is a *ten*; says so voluntarily or under questioning. Complete understanding with respect to both ones' and tens' columns.

- $0_2$  = Clear evidence of understanding of both adjustments (for ones and tens), but unable to explain so well as in  $0_1$ . Still, performance not mechanical or semimechanical.
- $0_3$  = Handles process semimechanically. Treats the "10" borrowed *as* a ten when dealing with the ones' figures, but as a "1" when dealing with the tens' figures. In D decreases the "top" figures, and in EA increases the "bottom" number without understanding that a *ten* is used.
- $0_4$  = Process mechanical, even if smooth and efficient. Borrows a "1," without thinking of it as a ten in case either of ones' or tens' column. With ones' column may set down the "1" beside the ones' figure thus creating, as it were, a fourth column and a new minuend. Treatment with regard to adjustment of tens' figure as mechanical as in  $0_3$ .
- $0_5$  = Utter confusion. Child may not realize that the subtrahend figure must be smaller than the minuend figure. Procedure is random and arbitrary.

## INTERVIEW RECORD

City \_\_\_\_\_ Teacher \_\_\_\_\_ Interviewer \_\_\_\_\_

| Subject        |   | 375<br>—268    |                | 797<br>—432    | 650<br>—437    |                | 532<br>—419    |                | 893<br>—673    | Total           |    |                                    |
|----------------|---|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|-----------------|----|------------------------------------|
|                |   |                |                |                |                |                |                |                |                | NB              | B  |                                    |
| Sarah<br>Smith | A | D              | C <sub>2</sub> | N <sub>1</sub> | D              | C <sub>2</sub> | D              | C <sub>3</sub> | N <sub>1</sub> | 2N <sub>1</sub> | 3D | 2C <sub>2</sub><br>1C <sub>3</sub> |
|                | B | O <sub>3</sub> |                |                | O <sub>3</sub> |                | O <sub>3</sub> |                |                |                 | 3  | O <sub>3</sub>                     |
| Bobby<br>Blake | A | D              | N <sub>5</sub> | N <sub>3</sub> | D              | N <sub>5</sub> | D              | N <sub>5</sub> | N <sub>3</sub> | 2N <sub>3</sub> | 3D | 3N <sub>5</sub>                    |
|                | B | O <sub>5</sub> |                |                | O <sub>5</sub> |                | O <sub>5</sub> |                |                |                 | 5  | O <sub>5</sub>                     |
|                | A |                |                |                |                |                |                |                |                |                 |    |                                    |
|                | B |                |                |                |                |                |                |                |                |                 |    |                                    |
|                | A |                |                |                |                |                |                |                |                |                 |    |                                    |
|                | B |                |                |                |                |                |                |                |                |                 |    |                                    |
|                | A |                |                |                |                |                |                |                |                |                 |    |                                    |
|                | B |                |                |                |                |                |                |                |                |                 |    |                                    |
|                | A |                |                |                |                |                |                |                |                |                 |    |                                    |
|                | B |                |                |                |                |                |                |                |                |                 |    |                                    |
|                | A |                |                |                |                |                |                |                |                |                 |    |                                    |
|                | B |                |                |                |                |                |                |                |                |                 |    |                                    |
|                | A |                |                |                |                |                |                |                |                |                 |    |                                    |
|                | B |                |                |                |                |                |                |                |                |                 |    |                                    |

Borrowing or carrying examples:

Row A first space: D or EA

second space: C or N, followed by 1, 2, 3,

or 4, to show degree of quality

Row B (degree of understanding)

O<sub>1</sub>, O<sub>2</sub>, O<sub>3</sub>, O<sub>4</sub>, or O<sub>5</sub>

Nonborrowing examples:

N<sub>1</sub>, N<sub>2</sub>, N<sub>3</sub>, or N<sub>4</sub>, to show quality of procedure

## APPENDIX C

### INSTRUCTIONS TO TEACHERS

#### EXPERIMENTAL PERIOD: GENERAL INSTRUCTIONS

##### *Major Purpose of Study*

The major purpose of the experiment is to discover the comparative advantages and weaknesses of two common but unlike procedures for borrowing in subtraction. According to the one procedure, Decomposition (D), one thinks for the example at the right, "Seven from 16, 9; 2 from 6, 4." According to the other procedure, Equal Additions (EA), one thinks, "Seven from 16, 9; 3 from 7, 4." We are investigating the two procedures from the standpoint of learning; that is, from the standpoint of their relative ease and difficulty to children who are just beginning to borrow. Accordingly, the first division of the experimental classes is into D- and EA-sections.

|     |
|-----|
| 76  |
| -27 |
| 49  |

##### *Minor Purpose of Study*

But both D and EA may be taught in different ways. Children may be taught "mechanically." In this case the thought process, as described above, is given "directly" to children at the outset of learning, without explanation but with plenty of drill to establish mastery. Or children may be taught "rationally." In this case the approach to the thought processes above is gradual. Children are first led to understand the procedure (whether D or EA) and are gradually guided to the shortened thought patterns described above. Accordingly, both D- and EA-sections are further divided into DM-, DR-, EAM-, and EAR-classes.

##### *Pre-experimental Period*

You have already received a bulletin which tells you what to do during the Pre-experimental Period prior to the two days of testing. Three additional suggestions are offered at this time:

(1) Be sure that before you start the experiment you have trained yourself in the borrowing procedure which you are to teach. Some of you habitually subtract by D; others, by EA. If you are to teach a different method, practice enough on the new procedure so that you can use it naturally and invariably. Your pupils must *never* see the procedure which they are not to learn—in the Pre-experimental Period, in the Experimental Period, or in the Postexperimental Period.

(2) If you are to teach a new method or are to teach a familiar method in a new way, try teaching it to a friend. Study the suggested plan of teaching in the bulletin "Special Instructions" and then have a go at it.

(3) Make use of Sets 1-6 of the Diagnostic Test on pages 97 and 98

of the textbook, as well as the corresponding practice sets (1-6) on pages 99 and 100, in order to be sure that your pupils have mastered subtraction without borrowing. We are not suggesting just when or how you should use these pages, but presumably you will do so after teaching pages 79-81 of the textbook and before the two days of testing.

### *Time Schedule for the Experiment*

We will give you a daily schedule for teaching the lessons. This schedule, which is part of your "Special Instructions," covers fourteen of the fifteen days of the Experimental Period. The one day is left for whatever purpose seems most important to you. In the experiment you will teach subtraction only through the skills developed on page 87 of the text, but you will of course make use of pages 88-100 also, as directed. Please plan to follow the schedule closely, at least so far as order of topics is concerned.

### *Extra Practice*

Attached to the bulletin "Special Instructions" are several sets of extra practice in computation, most of it subtraction with borrowing. These sets are keyed to the pages of the textbook and are to be used in connection therewith, *only as needed*. That is to say, this extra practice is given you in case the text does not provide all that you need. You are not to assign these sets just because you have them. If learning seems to be progressing satisfactorily, then use none of the extra practice, or as little as you want.

Don't hesitate to provide as much drill or practice as is needed on the basic subtraction facts. One of the commonest sources of error in subtraction is, as you know, uncertainty with the combinations. While no place is made specifically in the schedule for this practice, introduce it when and as often as you deem wise.

### *Evaluation of Learning*

The program of evaluation, except for your diaries or logs (described below), is entirely our responsibility. We shall provide and score tests, make arrangements for and administer interviews, etc. The explanation in this section is intended therefore only for your general enlightenment, in order to let you see what we shall do.

The relative merits of D and EA will be determined by the application of two criteria: (1) mastery of the process of borrowing (skill in borrowing) and (2) evidence of meaning (the degree to which children show that they understand what they do when they borrow).

Measures of comparative mastery (1) will be obtained from written tests given at the end of the Pre-experimental Period (two days), at the end of the experiment, and probably twice during the Postexperimental Period. The critical measures will be those (a) of rate and (b) of accuracy of work.

Measures of comparative meaning (2) will be secured partly from written tests but chiefly from interviews with individual children. There

will probably be two interviews with each child, one at the end of the experiment proper, one during the Postexperimental Period.

### Homework

No homework involving subtraction should be assigned during the Experimental and Postexperimental Periods. Children must be protected from borrowing procedures other than those they are learning.

### Teachers' Logs or Diaries

As noted above, the one phase of evaluation for which you are responsible is the report of your experiences in teaching. We are setting aside one section of this bulletin to talk about these logs because we know how important they are.

We suggest that you purchase a small notebook for a nickel or dime and that you use it exclusively to record what happens in your class from day to day. Label the pages "First Day," "Second Day," and so on; and state the page of the textbook you were teaching that day. (This plan is to make logs comparable. "Nov. 10" might mean one thing for one teacher, another thing for a different teacher.)

For each day tell us precisely what happened, e.g., what difficulties you encountered in teaching; how general these difficulties were (whether limited to one or two children, or characteristic of half or more); the nature and number of questions asked by your pupils; their errors, also by whom and by how many made, etc., etc. If everything proceeds smoothly and no difficulties develop, we need to know that fact as much as we need to know about problems.

Decide to keep these logs in a systematic fashion, putting down each day's happenings at the end of the arithmetic period or certainly that evening (you can't remember any longer). We don't think that you will find this job too troublesome, and you may find it most interesting. You may, for instance, decide that you would do another time in place of what you did this time. If so, tell us about it, and why.

The logs will be collected at the end of the study, and on the basis of experience we can predict that they will give us most helpful information. One caution: In your eagerness to "help" us *don't manufacture evidence!* (We know, that sounds bad!) This kind of "help" can only mislead us. Instead, record only what actually transpired each day; but tell us fully what did happen, so that we, who were not there, will know just what took place.

### SPECIAL INSTRUCTIONS: DR CLASSES

As has already been explained, your pupils will *eventually* think for the example at the right: "Eight from 15, 7; 2 from 5, 3," and write the remainders of 7 and 3 in their proper places as 37. *But* your pupils will not employ this verbal pattern (thought process) at first. Instead, they will approach this short, "direct" verbal pattern by passing through a series of steps in their thinking.

|     |
|-----|
| 65  |
| -28 |
| 37  |

This means that your progress at first will be slow. Like the teachers of DM and EAM classes, you will have only fifteen days for the experiment proper. Like them, too, you will have definite assignments for practically every day; but unlike them, you may want to take two days (and so, use the "extra" day which they will have) for the first lesson, in order to develop the meanings which your pupils are supposed to have. Don't hesitate to spend extra time at first; it may prove to be time well spent in the end.

The rationale of borrowing in the type example on page 82, which is reproduced above, is that we take one of the tens in 65 to change the 5 ones of 65 to 15 ones, thus leaving 5 tens. In other words, we use the idea that  $65 = 6 \text{ tens} + 5 \text{ ones}$ , or  $5 \text{ tens} + 15 \text{ ones}$ .

The lesson plan outlined below is intended to make the procedure of borrowing intelligible to your pupils by utilizing what they know about tens and ones. The first lesson may seem to be cumbersome, but we shall see. Indeed, it is precisely at this point that some of the best comparisons between R- and M-procedures may come to light. We shall want to know whether the slow, meaningful development at first pays dividends later in terms both of quality of computation and of understanding.

### *First and Second Days*

If you can attain the objectives of the first "lesson" in one day, well and good; but don't feel that you *must* complete it in one day. The schedule is written as if you may need two days, and so the "extra day" mentioned earlier may be lost to you. That need not bother you; you will be using the "extra" day earlier to good advantage.

Teach the material on page 82 of the textbook, but not in the manner there set forth. Instead, make use of whatever meanings your pupils have to "put sense" into the process of borrowing. It is suggested that you start with *Step 1* as this is described below; but if you find that your pupils cannot understand what you are trying to teach them, use *Preliminary Steps a* or *b*, or both of them. You will find descriptions of these preliminary steps at a later point.

|                |                                                          |                                                                                                          |                                                 |
|----------------|----------------------------------------------------------|----------------------------------------------------------------------------------------------------------|-------------------------------------------------|
| <i>Step 1.</i> | (1)                                                      | (2)                                                                                                      | (3)                                             |
|                | 65 = 6 tens + 5 ones,                                    | or 5 tens + 15 ones                                                                                      | $\begin{array}{r} 56 \\ 15 \end{array}$         |
|                | $-28 = 2 \text{ tens} + 8 \text{ ones},$                 | $\begin{array}{r} 2 \text{ tens} + 8 \text{ ones} \\ \hline 3 \text{ tens} + 7 \text{ ones} \end{array}$ | $\begin{array}{r} -28 \\ \hline 37 \end{array}$ |
|                | $\begin{array}{r} ? \quad ? \quad + \quad ? \end{array}$ |                                                                                                          |                                                 |

The development above is to be written on the board, first the abstract example itself; then (1) above, which is already familiar. Discuss the difficulty of subtracting the ones in (1), and suggest (or have the children suggest) the possibility of using (2). (They are to some extent at least familiar with this way of writing 65.) In (2) they work out the answer. Then rewrite the abstract example as shown in (3), with the figures changed. This is of course a "crutch," but let us worry about that. The crutch is a "number picture" which shows clearly, but in short form, what is done more fully in (2).

Repeat Step 1 with several other examples—1, 8, and 15 in the textbook, each time ending with a picture as in (3) and explaining always

in terms of tens and ones. Then have the pupils themselves write out (2) and (3) for several examples, such as  $42 - 23$ ,  $37 - 18$ , and the like. When you are satisfied that Step 1 is clear to all your pupils, move on to Step 2.

*Step 2.* Write the examples at the right on the blackboard. Have the pupils copy them and solve them, *using the crutch*. (You won't

$$\begin{array}{r} 52 \\ -36 \\ \hline \end{array} \qquad \begin{array}{r} 40 \\ -31 \\ \hline \end{array} \qquad \begin{array}{r} 35 \\ -16 \\ \hline \end{array} \qquad \begin{array}{r} 54 \\ -38 \\ \hline \end{array}$$

call it a crutch; you will simply tell the children to write the "number picture" to show what they are doing.) At the same time, help them to develop the following verbal pattern, which they will use with all four examples:

"I cannot take 6 from 2 (first example above); so, I borrow a ten from the 5 tens.

"I cross out the 5 and write a little 4 to show that I borrowed a ten.

"I write a little '1' in front of the 2 to show that I now have 12 instead of 2.

"Then I subtract: 'Six from 12 is 6,' and I write 6 in the ones' place; 'Three from 4 is 1,' and I write 1 in the tens' place."

*Preliminary Step a. Grouped objects.* Have each pupil "set up" 65, the minuend for the type example, with 6 bundles of ten sticks each and 5 extras. Have him take away the 8 (ones) first. Lead the children to see that they do not have enough ones (only 5), but that they can get enough by "borrowing" a ten-bundle and combining the 10 with the 5 ones. Then let them take away the 8 (ones); then the 2 ten-bundles (from the remaining 5 ten-bundles). Do this with two or three other examples ( $42 - 23$ ,  $37 - 18$ , for example) until the meaning is clear to all.

*Preliminary Step b.* This step is possible only for teachers who have been accustomed to present two-place numbers with symbols other than the figures. If you have not done this, go from Preliminary Step a above directly to Step 1.

*Pictured numbers.* Have the children draw 65  $\Phi \Phi \Phi \Phi \Phi \Phi \text{||||}$ . Then have them try to take away the 8 ones of 28. When they see that they cannot subtract, show them how to borrow a  $\Phi$  and combine the 10 ones so obtained with the 5 ones of 65. They have five  $\Phi$ 's left, from which they take two  $\Phi$ 's. Relate these activities to abstract subtraction; give practice with other examples; then go to Step 1.

There is no point in having your pupils learn this verbal pattern (or thought pattern) unless they understand what it means. Hence, do not hurry them to memorize the formula or pattern.

Assign the rest of the examples on page 82 for seatwork, telling the children to "draw the number picture" to show what they do when they subtract. In other words, insist that the crutch be used in *every* example with borrowing. Then correct the errors which appear on the papers—and by "correct" we mean not simply to have the children change wrong figures. We mean to go through the pattern of thinking with the child until he himself discovers the *reason* for his error.

If you want to stop at this point for extra practice, make use of Set 1, which has been prepared for this page. Here as elsewhere the extra practice is optional, to be used in accordance with your judgment.

Your log reports for this first lesson will obviously be of the greatest

importance to us. Reread, if you need to, the suggestions in "General Instructions" for taking notes and making records. But remember: Don't manufacture evidence just to be helpful.

### Third Day

Teach page 85 (you will omit pages 83 and 84). Use the type examples on the page for demonstration purposes, as well as examples 1, 6, 11, 16, and 21. Do not go back to Step 1 (see above) unless it is necessary. Rather, try to base the lesson on Step 2, with the explanation derived from the crutch. Insist on use of the verbal pattern developed in the first lesson, and have the children tell what they do when they borrow (the exact words of the patterns are not essential). Then assign the rest of the examples on page 83 for seatwork (no homework), the crutch ("number picture") being required with each solution where there is borrowing. If you want extra practice at this point, you will find it in Set Z, which contains the same kinds of examples as appear on page 85 of the text.

### Fourth Day

Teach page 86. Start the lesson with the three type examples given, and with examples 1, 6, 11, 16, and 21, as needed. The rest of the examples are to be used for seatwork, and the crutch must be shown in all work.

In teaching page 86, start again with Step 2. If you are confident that your pupils understand this step, move on to Step 3 below. But do not force the children to adopt the shorter verbal pattern unless they are ready for it. Instead, continue with the longer form of Step 2 for another day or so, shifting gradually to the shorter form. There is no hurry about this business; here as everywhere meaning is the important consideration.

*Step 3.* For type example (1), the children are to say merely, "Seven from 60 is 53"; for type example (2), "Nine from 16 is 7; 3 from 3 is 0; 2 from 3 is 1"; for type example (3), "Eight from 15 is 7; 2 from 2 is 0; 4 from 4 is 0." They will use this shorter verbal pattern, however, only with the examples rewritten on the blackboard *with the crutch*

$$\begin{array}{r}
 \begin{array}{r}
 \text{(1)} \\
 \begin{array}{r}
 \overset{5}{8} \overset{1}{0} \\
 - \quad 7 \\
 \hline
 5 \ 3
 \end{array}
 \end{array}
 \qquad
 \begin{array}{r}
 \text{(2)} \\
 \begin{array}{r}
 \overset{2}{3} \overset{3}{4} \overset{1}{6} \\
 - \ 2 \ 3 \ 9 \\
 \hline
 1 \ 0 \ 7
 \end{array}
 \end{array}
 \qquad
 \begin{array}{r}
 \text{(3)} \\
 \begin{array}{r}
 \overset{4}{2} \overset{3}{8} \overset{1}{5} \\
 - \ 4 \ 2 \ 8 \\
 \hline
 \quad \quad 7
 \end{array}
 \end{array}
 \end{array}$$

*as shown* here. Don't fail to write the examples with the crutch, for the pupils are still to think, in short form of course, in terms of what they do when they borrow in subtracting.

Set 3 will provide any needed practice. In correcting all seatwork be sure that you get beneath the mistake (of whatever character) to find out the reason therefor.

### Fifth Day

Teach page 87, using the type examples rewritten with the crutch, as well as examples 1, 6, 11, and 16. Use Step 2 only as necessary, and urge the adoption of the short form in Step 3 as rapidly as is consistent with sound progress.

When you assign the other examples on the page for seatwork, insist upon use of the crutch. Do the same with any extra practice (for which Set 4 has been prepared). Here as elsewhere, the extra practice is to be used only when you think it should be used.

*Note:* Collect the papers of your pupils after they have worked the examples on page 87 and turn them over to your co-ordinator. They will be forwarded to Duke University, where we shall want to check use of the crutch. In assigning this particular body of seatwork, *do not be any more insistent than usual* that the crutch be used.

#### *Sixth Day*

Pages 88 and 89 constitute a single day's work. If you do not want to assign all the problems for seatwork, treat some of them in the class period. Whenever borrowing occurs, require use of the crutch and urge the verbal pattern of Step 3. Set 5 of extra practice may be used at this point.

#### *Seventh Day*

Use page 90. The crutch and Step 3 are to be required in case of borrowing. Set 6 supplies extra practice.

#### *Eighth Day*

Page 91: crutch; Step 3 only (we hope, by this time). However, if some pupils must still use Step 2, tell us about them in the log, but let them use it. For extra practice, use Set 3, 4, 5, or 6, whichever you have not assigned.

#### *Ninth Day*

The problems on pages 92 and 93. All borrowing with the crutch, and reports of borrowing according to Step 3. Set 7 provides extra practice, if needed.

#### *Tenth Day*

Examples at the bottom of page 93; crutch; Step 3. Extra practice, Set 8.

#### *Eleventh Day*

Problems on pages 94 and 95; crutch, Step 3. Extra practice, Set 9.

#### *Twelfth Day*

Page 95, bottom; thirty-six examples; crutch; Step 3.

#### *Thirteenth Day*

Problems on page 96; crutch; Step 3. Extra practice: Use Set 7, 8, or 9, whichever you have not assigned.

#### *Fourteenth and Fifteenth Days*

Use Sets 6, 7, 8, 9, and 10 on page 98 as a test, and assign the corresponding sets of practice material as needed: Sets 6, 7, 8, 9, and 10 on page 100. In all this work ask for the crutch on the papers.

*Note:* The test papers are to be sent to your co-ordinator, who will forward them to Duke University.

## SPECIAL INSTRUCTIONS: DM-CLASSES

As has already been explained, for the example at the right, you are to *give your pupils at the outset the thought patterns* or verbal patterns they are to use, and you are never to offer any explanation beyond that on page 82 of the textbook. By "never" we mean precisely that. You won't explain the first day, and you won't explain at any later time (until after the Post-experimental Period).\*

|     |
|-----|
| 65  |
| -28 |
| 37  |

If the children ask questions which call for an explanation of any step in borrowing, answer them with "explanations" which explain nothing at all. For example, if a child, in connection with the example above, asks the question, "But why do we change the 5 to 15?," answer, "Because we cannot take 8 from 5," or "Whenever we subtract a larger number from a smaller number, we have to make the smaller number bigger." Or, suppose a child asks, in connection with the second subtraction above, "But why did we make the 6 a 5?," answer, "Whenever we make one number larger, we make the next one smaller." In a word, use "explanations" which *seem* to explain but which actually provide the child with no more than half-answers.

In the end, your teaching will really consist in telling the children what to think, in seeing that they master the verbal pattern or formula you give them, and in giving drill in thinking that formula as they work both the demonstration examples in class and the examples assigned for seatwork. (*Remember: no home work.*)

*First Day*

Use page 82 of the textbook, together with the suggestions there given. Follow the procedure sequence below.

- (1) Point out the new element to be learned, and show how the new work differs from examples previously studied.
- (2) Demonstrate the pattern of solution, emphasizing the steps in thinking. (See below for the pattern or formula.)
- (3) Have some child repeat the solution pattern. ("Who thinks he can do this example as I have done it?")
- (4) Repeat with a second child; a third.
- (5) Ask such questions as the following (for the example above): "How did we get the 3 [point to it] in the remainder?"; "How did we get the 7 [point] in the remainder?" In each instance see that the child repeats the appropriate line in the verbal pattern or formula.
- (6) "Now let's see if we can do another example of this kind," and so on, as long as needed.

The first verbal pattern, to be superseded by a shorter one toward the end of the period, is:

\* You may take out your indignation on us in the log, along with a description of the incidents that aroused it. But, to repeat a warning: Don't manufacture evidence. For instance, don't stimulate the children to ask "Why?" And if they don't ask, "Why?," tell us that. It is important for us to know.

"I can't take 8 from 5, so I think of 5 as 15.

"Then I think, 'Eight from 15 is 7,' and I write 7.

"Since I thought of 5 as 15, I must think of 6 as 5.

"Then I think, 'Two from 5 is 3,' and I write 3."

Use this verbal pattern for the type example on the page,  $65 - 28$ , and then with examples 1, 8, and 15.

Next, shorten the verbal pattern to: "Eight from 15, 7; 2 from 5, 3."

Assign the other examples on the page for seatwork, and then examine the papers to detect errors. Correct the work by having the children apply the longer or the shorter verbal pattern above.

For extra practice, if you want any, use Set 1 on the page attached to this bulletin.

### Second Day

Teach page 85 of the textbook. (You will omit pages 83 and 84 since they present different methods of borrowing.) And teach this lesson exactly as shown in the book. Use the longer verbal pattern at first if this seems desirable, as for the first type example which is reproduced at the right: "I can't take 6 from 4; so I think of 4 as 14; then, 6 from 14 is 8, and I write 8. Since I thought of 4 as 14, I must think of 7 as 6; then 2 from 6 is 4, and I write 4. Then, 2 from 4 is 2, and I write 2."

|             |
|-------------|
| 474         |
| <u>-226</u> |
| 248         |

Then have the children use the shorter form: "Six from 14, 8; 2 from 6, 4; 2 from 4, 2."

Take the same two steps (if necessary) for the second type example in the text,  $965 - 49 = 916$ .

Use examples 1, 6, 11, 16, and 21 for demonstration purposes in class, and assign the rest of the examples on the page for seatwork.

Extra practice (if needed), Set 2.

*Remember:* No explanations; no variations from the suggested verbal patterns or formulas. Otherwise you introduce an experimental variable which ruins your data.

### Third Day

Teach page 86 according to the text. Use the longer thought pattern only for children who have not mastered it, and urge adoption of the shorter form, as for  $435 - 428$ : "Eight from 15, 7; 2 from 2, 0; 4 from 4, 0."

Use examples 1, 6, 11, 16, and 21 for demonstration purposes (as many as needed), and assign the rest of the examples for seatwork. For extra practice, Set 3.

*Remember:* You are allowed one extra day in the experimental period. Use it when and as you wish, within the limits of your method of teaching borrowing.

### Fourth Day

Teach page 87 of the text, following the directions given for the two type examples at the top of the page. Only if necessary, use the longer verbal pattern, but plan to use it no longer. In other words, have the children from now on use only the shorter and more "direct" form of

thinking. Employ examples 1, 6, 11, and 16 for the purposes of demonstration, and assign the other sixteen examples for seatwork. Extra practice (if needed), Set 4.

#### *Fifth Day*

Every day (without further mention on our part) correct the work assigned the previous day, and do so by having children repeat the formula, or that part of it which applies.

Assign the problems on pages 88 and 89 for one lesson. If there are too many for seatwork, then deal with part of them in class. Extra practice is Set 5; it includes the kinds of subtraction example taught in the first four lessons, plus a few examples without borrowing. Insist on the use of the short formula exclusively.

#### *Sixth Day*

Assign the verbal problems on page 90, and use Set 6 of the extra practice, unless you need the extra time for something else.

#### *Seventh Day*

Teach page 91.

#### *Eighth Day*

Teach the eleven problems on pages 92 and 93. Use Set 7 of extra practice unless you need to do something else.

#### *Ninth Day*

Assign the sixteen examples at the bottom of page 93. For extra practice, use Set 4, 5, or 6—whichever you have not yet assigned.

#### *Tenth Day*

Assign the twelve problems on pages 94 and 95, together with extra practice from Set 8, if this is needed.

#### *Eleventh Day*

Assign the thirty-six examples at the bottom of page 95.

#### *Twelfth Day*

Work with the problems on page 96, and use Set 9 of extra practice if needed.

#### *Thirteenth and Fourteenth Days*

Use Sets 7, 8, 9, and 10 of the Diagnostic Test on page 98 *as a test*. Then assign practice as needed from Sets 7, 8, 9, and 10 on page 100. Note that this practice is keyed to corresponding sections of the test. Practice material should be assigned in terms of errors made on the test.

#### *Fifteenth Day*

Nothing definite is assigned for the fifteenth day. In all probability you will have made use of this day already, postponing the assignments after this free day; as, for example, the fifth day's assignment to the

sixth day, the sixth day's to the seventh, and so on. Otherwise, use the class period as seems best to you, perhaps having the children work some set of extra practice which you omitted.

### SPECIAL INSTRUCTIONS EAR CLASSES

As has already been explained, your pupils will *eventually* think for the example at the right: "Eight from 15, 7; 3 from 6, 3," and will write the remainders of 7 and 3 in their proper places as 37. But your pupils will not employ this verbal pattern (thought process) at first. Instead, they will approach this short, "direct" verbal pattern by passing through a series of steps in their thinking.

|     |
|-----|
| 65  |
| -28 |
| 37  |

The rationale of subtraction in the type example on page 83, which is reproduced above, is that we add *one ten* to both minuend and subtrahend, in order to permit the subtraction of the ones. That is, we add one ten as 10 ones to the 5 (ones) of 65 to get 15, from which we may take the 8 (ones) of 28; and then, in order to "balance accounts," we add a ten to the 28, this time to the 2 tens; and so we subtract 3 tens from 6 tens.

Meaningful mastery of this kind of subtraction—and that is what you are to try to develop—requires that your pupils learn (1) a new and important mathematical principle and (2) its special application to borrowing in subtraction. The new principle (1) is this: The difference between two numbers is not altered if the same amount is added to both numbers. The application (2) obviously consists in adding a ten to both minuend and subtrahend when carrying is to be done. Both principle and application must be taught, and must be taught at the outset of the experimental period. For this reason your progress will be slow at first.

Don't let it bother you if you don't get ahead as rapidly as you might like in the first days of the experiment. Two days are assigned for "the first lesson"; we think you will need that many days. You may need another day. If so, don't hesitate to take it. The schedule is flexible, and later on you may be able to combine the "lessons" for two or more days into a single lesson. (You can do this readily by omitting some of the suggested extra practice—and you may find that this extra practice is not needed.) The essential fact is that your pupils understand what they do, and they should not be led along to new skills until they have intelligent mastery of the first ones.

Some of the crucial data for this experiment will come from your log reports in the first days of the experiment. It will be of interest, for example, to see whether the slow rate of development at the start does not later pay large dividends in quality of work and in understanding. It will be of interest, too, to learn just how difficult it is to rationalize the method of EA—and only your log notes can tell us. For these reasons, and others, we are especially anxious that you make full and complete records of your teaching experience.

*First and Second Days*

If you can teach all that is outlined for this "lesson" in one day, well and good. But don't feel that you *must* complete the unit in a single day, or even in two days for that matter. Remember: You have two important understandings to teach. A suggested method is outlined below. Study it carefully; try it out on someone, and make whatever modifications appear to be necessary. But teach the understandings!

The outline for the unit takes a great deal of space. From this fact do not conclude that the task of teaching is impossible. Once you begin teaching, you will discover that it is not impossible. The account is long merely because many words are needed to set forth all the aspects of the undertaking. Again let us recommend the practice of perfecting your explanations by trying them on someone before you face your class.

(1) *New mathematical principle.* THE DIFFERENCE BETWEEN TWO NUMBERS IS NOT CHANGED IF THE SAME AMOUNT IS ADDED TO BOTH NUMBERS.

This principle can be developed by using such series of relationships as

A and B at the right. For this purpose, employ: (a) actual objects, (b) drawings of numbers, (c) simple, easily

imagined verbal problems, and (d) series of abstract examples as illustrated. In the end we want the children to understand the principle, whether or not they can put it into the formal language in which it is stated above. But be sure that they can state the principle in some way, because the verbal statement will summarize their learning.

If you use *actual objects*, have the child lay out 4, then take away 1, and find he has 3 left. Then have him increase both groups by 10 ( $4 + 10$ , and  $1 + 10$ ) and repeat the subtraction. Continue in this way through the series. If you use *drawings*, the first two items in series A are: (1)  $|||| - |$  and (2)  $\Phi |||| - \Phi |$ . If you use *verbal problems*, construct a story something like this: "Joe had 4 pennies; Tom had 1 penny. How much more had Joe than Tom? Their fathers gave them each a dime. Then Joe had how much? And Tom had how much? Then how much more had Joe than Tom?" If you use *abstract series* (and you should, after first using one or more of the other three devices just mentioned), write the first item on the board, and construct the series with the children's help. With *all* devices stress the idea contained in the principle, ending, as has been suggested, with some verbal formulation of the principle.

(2) *Application of the principle to borrowing in subtraction.* Don't take up the second understanding until you are *sure* that your children have grasped the first. Then teach the application of the principle according to the steps below. There are three steps, only two of which are developed during the first three or four days. Start with Step 1 if

| A                 | B                 |
|-------------------|-------------------|
| (1) $4 - 1 = 3$   | (1) $9 - 7 = 2$   |
| (2) $14 - 11 = 3$ | (2) $19 - 17 = 2$ |
| (3) $24 - 21 = 3$ | (3) $29 - 27 = 2$ |
| (4) $34 - 31 = 3$ | (4) $39 - 37 = 2$ |

you can. If you find your pupils confused by it (or if, without trying it, you know they would be confused), then make use of *Preliminary Steps a* and *b*, which are given below.

Start your instruction with page 83 of the textbook, omitting page 82. Use the type example ( $65 - 28$ ) and the ideas given on the text page, but supplement them as shown in the following paragraphs.

### Step 1.

|     |                        |     |                 |                  |                 |                                      |
|-----|------------------------|-----|-----------------|------------------|-----------------|--------------------------------------|
| 65  | 65 = 6 tens + 5 ones,  | (1) | or              | 6 tens + 15 ones | (2)             | (3)                                  |
| -28 | -28 = 2 tens + 8 ones, | or  | 3 tens + 8 ones | 3 tens + 7 ones  | 3 tens + 7 ones | 6 <sup>1</sup> 5<br>3 <sup>2</sup> 8 |
| ?   | ?                      | ?   |                 |                  |                 | 3 7                                  |

First, copy the example on the blackboard, and point out the new element in subtraction. As a start toward the solution, rewrite the example as shown in (1). The difficulty of subtracting the ones will at once be apparent. Suggest (2) as the way out—or, much better, try to lead the children to suggest the idea themselves. (After all, it follows from the principle which they have just learned—all except the reason for adding a ten to each number.) The reason why we add a ten and not some other number is that if we add anything less we should have to add that number to the ones' figure of the subtrahend, and we should still be unable to subtract. For example, if we add 9 to each ones' figure, we should have to subtract 17 from 14, and the dilemma would remain. By adding a ten, we can combine 10 ones with the ones' figure of the minuend and a ten with the tens' figure of the subtrahend. (*Note:* This explanation is for *you*. Don't try it on any child unless he is a very, very superior child who will not be confused.)

*Preliminary steps.* Steps *a* and *b* are intended to provide meanings more concretely than can be done with abstract numbers. Study the suggested activities, and if you feel that they would confuse your pupils or take a disproportionate amount of time, act accordingly; use one step, or neither.

*Preliminary Step a. Grouped Objects.* You can readily demonstrate borrowing by using grouped objects. Have the children "set up" the two numbers 65 and 28, with bundles of ten sticks and extras (thus,  $65 = 6$  ten-bundles, plus 5 extras;  $28 = 2$  ten-bundles, plus 8 extras). Have them keep the ten-bundles in line, and the extras in line. Then, lead them to see that there are more extras (more "ones") in the lower number, so that no subtraction can be made. On the other hand, if they put a ten-bundle (broken up into 10 ones) with the 5 ones of 65 and at the same time put a ten-bundle with the 2 ten-bundles of 28 (thus making 3 ten-bundles), they will be able to subtract:  $15 - 8, 6 - 3$ . Repeat with several examples, and later on relate this activity to the abstract work on Step 1.

*Preliminary Step b. Drawings of Numbers.* Have the children draw pictures for 65 and 28, as at (1) below. Next have them compare the ones in the two numbers and see that the subtraction cannot be made—8 from 5. Then have them add a ten to each number, breaking it up into 10 ones for the 65, but placing it with the other  $\Phi$ 's in 28. The subtractions can now be made. Repeat with other examples.

(1)  $\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi - \Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi$

(2)  $\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi - \Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi\Phi$

From (2) move on to (3), which is really a "number picture" to show what has been done. Yes, it is a "crutch," but let us worry about that. You teach it, and we will answer for the consequences.

Next, present another borrowing example, e.g.,  $43 - 17$ . Write it on the blackboard in abstract form, then in forms (1), (2), and (3). Have the children tell why form (1) doesn't help any, and why form (2) does help. When you write form (3), have some child analyze the "picture" to show that it really is the same as (2).

Then, have the children themselves write out the solutions of several borrowing examples, using forms (2) and (3). You may use these examples:  $37 - 19$ ;  $53 - 27$ ;  $44 - 26$ . When they have arrived at form (3) for each example, let them analyze it orally for you and for the class.

*Step 2.* Move on to Step 2 only when your children thoroughly understand Step 1. Step 2 is intended merely to give the children a way to think about borrowing. For the type example,  $65 - 28$ , they are to think:

"I can't take 8 from 5; so I add 10 to 5 and get 15.

"Now I can subtract the ones—'Eight from 15, 7'; and I write 7 in the ones' place.

"Since I added 10 to the ones' figure in the top number, I add a ten to the tens' figure in the bottom number, and 2 becomes 3.

"Now I can subtract the tens—'Three from 6, 3'; and I write 3 in the tens' place."

As you see, this verbal pattern merely translates into words the drawing in the crutch; that is its purpose. But there is no point in having the children memorize the formula if they do not understand it. One way to discover whether they *do* understand is to have several children, one after another and using different examples written in crutch form, go through the formula, at the same time pointing to the numbers that are involved.

When you feel certain that your pupils have complete understanding and know the formula, assign the rest of the examples on page 83 for seatwork (*no homework*). Insist that the crutch (or "the number picture") appear in every solution where there is borrowing. When the children explain their work, have them use the crutch form and point to the numbers, original and changed. When you correct their work, have them follow through the formula and themselves detect the *reasons* for their errors.

If you want to stop for a time after teaching Steps 1 and 2, use the extra practice in Set 1, which contains examples of the kind taught on page 83.

### Third Day

Teach page 85 (you will omit page 84). Use the type examples on the page for demonstration purposes, as well as examples 1, 6, 11, 16, and 21. Do not go back to Step 1 (see above) unless it is necessary. Rather, try to base the lesson on Step 2, with the explanation derived from the

crutch. Insist on use of the verbal pattern developed in Step 2, and have the children tell what they do in borrowing (the exact words are not of course essential). Then assign the rest of the examples on page 85 as seatwork, the crutch ("number picture") being required with each solution where there is borrowing. If you want extra practice at this time, you will find it in Set 2, which contains the kinds of examples which appear on page 85 of the text.

#### Fourth Day

Teach page 86. Start the lesson with the three type examples given, and with examples 1, 6, 11, 16, and 21 as needed. The rest of the examples are to be used for seatwork, and the *crutch* must be shown in all work.

In teaching page 86, start again with Step 2. If you are confident that your pupils understand what they are doing, you may then move on to Step 3, which shortens the verbal pattern. But *do not force* the children to adopt the shorter verbal pattern unless they are ready for it. Instead, continue with the longer form of Step 2 for another day or more, shifting gradually to the shorter form. There is no hurry about this business: meaning remains here as everywhere the important consideration.

Step 3. Type examples (1), (2), and (3) are rewritten below with the crutch. When the students take Step 3 they will continue to work with and from the crutch, but they will say: for (1), "Seven from 60 is 53"; for (2), "Nine from 16 is 7"; 4 from 4 is 0; 2 from 3 is 1"; for type (3), "Eight from 15 is 7; 3 from 3 is 0; 4 from 4 is 0." These statements are "the shorter form" mentioned above. They are appropriate for the crutch, of course, in terms of which the children are to continue to think.

$$\begin{array}{r} \text{(1)} \quad 60 \\ - 7 \\ \hline 53 \end{array} \quad \begin{array}{r} \text{(2)} \quad 3 \ 4 \ 16 \\ - 2 \ 4 \ 9 \\ \hline 1 \ 0 \ 7 \end{array} \quad \begin{array}{r} \text{(3)} \quad 4 \ 3 \ 15 \\ - 4 \ 3 \ 2 \ 8 \\ \hline 7 \end{array}$$

Set 3 will provide any needed practice. In correcting work be sure to get beneath the mistake (of whatever character) to find the reason therefor.

#### Fifth Day

Teach page 87, using the type examples rewritten with the crutch, as well as examples 1, 6, 11, and 16. Use Step 2 only as necessary, and urge the adoption of the short form in Step 3 as rapidly as is consistent with sound progress.

Then assign the other examples on the page for seatwork, insisting on use of the crutch. Do the same with any extra practice (for which Set 4 has been prepared). Here as elsewhere, the extra practice is to be used *only* if needed.

*Note:* Collect the papers of your pupils after they have worked the examples on page 87 and turn them over to your co-ordinator. They are to be forwarded to Duke University, where we shall want to check on

use of the crutch. However, in assigning the seatwork insist no more than usual upon use of the crutch.

### *Sixth Day*

Pages 88 and 89 constitute a single lesson. If you do not want to assign all the problems for seatwork, treat some of them in the class period. Whenever borrowing occurs, require the crutch and urge use of the verbal pattern of Step 3. Set 5 provides extra practice.

### *Seventh Day*

Page 90; require crutch and Step 3 pattern. Set 6 for extra practice as needed.

### *Eighth Day*

Page 91; crutch; Step 3 only (we hope, by this time. However, if some children must still remain at Step 2, permit them to, but tell us about them in your log). For extra practice use Set 3, 4, 5, or 6, whichever you have not assigned.

### *Ninth Day*

Pages 92 and 93, the problems; crutch; Step 3; extra practice, Set 7.

### *Tenth Day*

Page 93, the examples; crutch; Step 3; Set 8 for extra practice.

### *Eleventh Day*

Pages 94 and 95, the problems; crutch; Step 3; Set 9.

### *Twelfth Day*

Page 95, bottom; examples; crutch; Step 3; Set 8 or 9.

### *Thirteenth Day*

Page 96, problems; crutch; Step 3; any unassigned set of practice.

### *Fourteenth and Fifteenth Days*

Use Sets 6, 7, 8, 9, and 10 on page 98 as a test, and assign the corresponding sets of practice material as needed: Sets 6, 7, 8, 9, and 10 on page 100. In all work require use of the crutch and of Step 3.

*Note:* Send the test papers to your co-ordinator, to be forwarded to Duke University.

### SPECIAL INSTRUCTIONS: EAM CLASSES

As has already been explained, for the example at the right, you are to *give your pupils at the outset the thought patterns* or verbal patterns they are to use, and you are never to offer any explanation beyond that on page 83 of the textbook. By "never" we mean precisely that. You won't explain the first day, and you won't explain at any later time (until after the Post-experimental Period.)\*

|     |
|-----|
| 65  |
| -28 |
| 37  |

\* You may take out your indignation on us in the log, along with a description of the incidents which aroused it. But, to repeat a warning: Don't manufacture evidence. For instance, don't stimulate the children to ask "Why?" And if they don't ask this question, tell us that. It is important for us to know.

If the children ask questions which call for an explanation of any step in borrowing, answer them with "explanations" which explain not at all. For example, if a child, in connection with the first subtraction in the example above, asks, "But why do we change the 5 to 15?" answer, "Because we cannot take the 8 from 5," or "Whenever we subtract a larger number from a smaller number, we have to make the smaller number bigger." Or suppose a child asks, in connection with the second subtraction above, "But why do we make the 2 a 3?" answer, "We made the 5 a 15; so we change the 2 to 3"; or, "If you change the 5, you have to change the 2, too"; or, "If you make the top number larger you have to make the bottom number larger also." In a word, use "explanations" which *seem* to explain but which actually provide nothing better than half-answers.

In the end your teaching will really consist in telling the children what to think, in seeing that they master the verbal pattern you give them, and in giving drill in thinking that formula as they work both the demonstration examples in class and the examples assigned for seatwork. (*Remember*: no homework.)

### *First Day*

Use page 83 of the textbook, together with the suggestions there given. Follow the procedure sequence below:

- (1) Point out the new element to be learned, and show how the new examples differ from those previously studied.
- (2) Demonstrate the pattern of solution, emphasizing the steps in thinking. (See below for the pattern or formula.)
- (3) Have some child repeat the solution pattern. ("Who thinks he can do the example as I have done it?")
- (4) Repeat with a second child; with a third.
- (5) Ask such questions as the following (for the example above): "How did we get the 3 [point to it] in the remainder?"; "How did we get the 7 [point] in the remainder?" In each instance see that the child repeats the appropriate sentence or phrase in the pattern or formula.
- (6) "Now, let's see if we can do another example of this kind," and so on, as long as needed.

The first verbal pattern, to be superseded by a shorter one toward the end of the period, is:

"I can't take 8 from 5; so I think of 5 as 15.

"Then I think, 'Eight from 15 is 7,' and I write 7.

"Since I thought of 5 as 15, I must think of 2 as 3.

"Then I think, 'Three from 6 is 3,' and I write 3."

Use this verbal pattern for the type example on the page, 65 — 28, and then for examples 1, 8, and 15.

Next, shorten the verbal pattern to: "Eight from 15, 7; 3 from 6, 3."

Assign the other examples on the page for seatwork, and then examine the papers to detect errors. Correct the work by having the children apply the direct or shorter verbal pattern above.

For extra practice, if you want any, use Set 1 on the page attached to this bulletin.

### Second Day

Teach page 85 of the textbook. (Omit pages 82 and 84, since they present different methods of borrowing.) Teach this lesson exactly as shown in the book. Use the longer verbal pattern at first if this seems desirable, as for the first type example at the right: "I can't take 6 from 4; so I think of 4 as 14; then, 6 from 14 is 8, and I write 8. Since I thought of 4 as 14, I must think of 2 as 3; then 3 from 7 is 4, and I write 4. Then 2 from 4 is 2, and I write 2."

|      |
|------|
| 474  |
| -226 |
| 248  |

Then have the children use the shorter form: "Six from 14, 8; 3 from 7, 4; 2 from 4, 2."

Take the same two steps (if necessary) for the second type example in the text,  $965 - 49 = 916$ .

Use examples 1, 6, 11, 16, and 21 for demonstration purposes in class, and assign the rest of the examples on the page for seatwork.

Extra practice (if needed), Set 2.

*Remember:* No explanations; no variations from the approved verbal patterns or formulas. Otherwise you introduce an experimental variable which ruins your data and makes them useless for this investigation.

### Third Day

Teach page 86 according to the text. Use the longer thought pattern only for children who have not mastered it, and urge adoption of the shorter formula, as for  $435 - 428$ : "Eight from 15, 7; 3 from 3, 0; 4 from 4, 0."

Use examples 1, 6, 11, 16, and 21 for demonstration purposes (as many as needed), and assign the rest of the examples for seatwork. For extra practice, Set 3.

*Remember:* You are allowed one extra day in the experimental period. Use it when and as you wish, within the limits of your method of teaching borrowing.

### Fourth Day

Teach page 87 of the textbook, following the directions given for the two type examples at the top of the page. Only if necessary, use the longer verbal pattern, but plan to use it no longer. In other words, have the children from now on use only the shorter and more "direct" form of thinking. Employ examples 1, 6, 11, and 16 for the purposes of demonstration, and assign the other sixteen examples for seatwork. Extra practice (if needed), Set 4.

### Fifth Day

Every day, without further mention on our part, correct the work assigned for the previous day, and do so by having children repeat the formula, or that part of it which applies.

Assign the problems on pages 88 and 89 for one lesson. If there are too many for seatwork, then deal with part of them in the class. Extra practice is Set 5; it includes all kinds of subtraction example taught in the first four lessons, plus a few examples without borrowing. Insist on the use of the short formula exclusively.

#### *Sixth Day*

Assign the verbal problems on page 90, and use Set 6 of the extra practice, unless you need the extra time for something else.

#### *Seventh Day*

Use page 91.

#### *Eighth Day*

Teach the eleven problems on pages 92 and 93. Use Set 7 of the extra practice unless you need to do something else.

#### *Ninth Day*

Assign the sixteen examples at the bottom of page 93. For extra practice, use Set 4, 5, or 6—whichever you have not yet assigned.

#### *Tenth Day*

Assign the twelve problems on pages 94 and 95, together with extra practice from Set 8, if this is needed.

#### *Eleventh Day*

Assign the thirty-six examples at the bottom of page 95.

#### *Twelfth Day*

Work with the problems on page 96, and use Set 9 of the extra practice if needed.

#### *Thirteenth and Fourteenth Days*

Use Sets 7, 8, 9, and 10 of the Diagnostic Test on page 98 as a test. Then assign practice as needed from Sets 7, 8, 9, and 10 on page 100. Note that this practice is keyed to the corresponding sections of the test. Practice material should be assigned in light of the errors made on the test.

#### *Fifteenth Day*

Nothing definite is scheduled for the fifteenth day. In all probability you will have made use of this day already, postponing the assignments after the free day; as, for example, the fifth day's assignment to the sixth day, the sixth day's to the seventh day, and so on. If not, use the class period on this extra day for whatever seems most necessary, perhaps having the children work some set of extra practice which you have not used.

#### INSTRUCTIONS FOR POSTEXPERIMENTAL PERIOD

Following the experimental period of three weeks is to come a "Post-experimental Period" of six weeks (instead of eight weeks as originally

planned). The purpose of the Postexperimental Period is to observe the significant changes, if any, which take place in understanding and use of subtraction with borrowing. These changes (again, if any) we want to be spontaneous—that is, changes which the children make themselves without prompting or suggestion or specific instruction.

In order to insure that changes will be spontaneous, it is necessary that:

- (1) children continue to use the method they have been taught, whether D or EA, and that they never see anything else,
- (2) no *new* explanations be given for whatever method has been taught (for DM and EAM teachers this statement is equivalent to saying that no explanation at all will be offered),
- (3) examples in subtraction be restricted to the types actually taught in the experiment.

With respect to (3), a check of the state textbook shows that no new type of subtraction is taught before page 189, and you won't reach this page until long after the Postexperimental Period. (This should be over by the middle of February.) If, however, you do not use the state textbook in your class, (1) notify your co-ordinator at once and (2) in conference with her see whether other types of subtraction may not be postponed until after the Postexperimental Period.

*In the case of DR and EAR teachers*, there is to be one important exception to the opening statements in this bulletin. You are to *deny* your pupils use of the crutch from the start of the Postexperimental Period onward. The procedure is to illustrate a "short way" to subtract without the device and to demonstrate that the crutch is really no longer needed. For a few lessons whenever you assign seatwork in subtraction remind the children not to use the crutch. If you find any pupils continuing to rely upon it, find out why and try to teach them away from it. We shall probably send you a test at the end of the third week, the papers on which will be examined solely to discover the prevalence of use of the crutch.

*All teachers*: Do not neglect your diaries! If anything significant happens during the Postexperimental Period, make a record in your diary. For EAR and DR teachers this is particularly important, as explained in the paragraph immediately above.

The Postexperimental Period will be concluded with a written group test and probably with interviews (if these can be arranged). *After* this testing you will be free to do whatever you may want to do by way of repairing any damages in the standing you have with your pupils!

#### FINAL INSTRUCTIONS

The Postexperimental Period will end six weeks after the final test is given in your class (that is, six weeks after the test which concludes the Experimental Period). Determine the exact date by consulting your co-ordinator.

Midway through the Postexperimental Period a test is to be given to

all *children in DR and EAR classes* (see directions below); and at the end of the Postexperimental Period the Retention Test is to be given to all *children* regardless of class (directions below).

As soon as is convenient after the Retention Test, please send your diary to your co-ordinator. Before doing so, however, glance through it and underscore items which you especially want us to note. Also, add any comments that may help us to understand your observations and point of view. It might not be a bad idea to conclude your diary with an evaluation of your experiences as a whole.

### *Postexperimental Test*

(for children in DR and EAR classes only)

The purpose of this test is to ascertain the status of the crutch, to learn the extent to which the crutch is still used (if at all). On this account the test papers will not be scored for rate or accuracy of work; instead, a count will be made of the number of times the crutch appears on each paper.

Obviously, the way you give the test directions may affect the use of the crutch; hence, be especially careful to *suggest nothing at all* in this respect. Say nothing either to encourage or to discourage its use. The safest plan is to say nothing at all about it and then to parry questions if they are asked.

### *Retention Test*

This test is to be given to all *children* six weeks after the test which ends the Experimental Period. Consult your co-ordinator for the date. The measures to be obtained are: (a) rate, (b) accuracy, and (c), in DR and EAR classes, use of the crutch.









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